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# **Multiple Ionization of Atoms by Heavy Ions**

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# ~~Independent Electron Model~~

**Simple alternative approach, many electron system, based on LPA**

**Simple processes: ionization, stopping, straggling,...**

**Complex systems: molecules, solids, surfaces...**

**Complex problems: multiple ionization, antiscreening, induced potential**

**Talk 1- collision with surfaces**

**2- moments: Straggling, stopping & ionization X sect.**

**3- Multiple ionization of atoms**

**4- Advantages & disadvantages**

**Stopping (PRA 69, 062903 (2004), 73, 024901 (2006) ).**

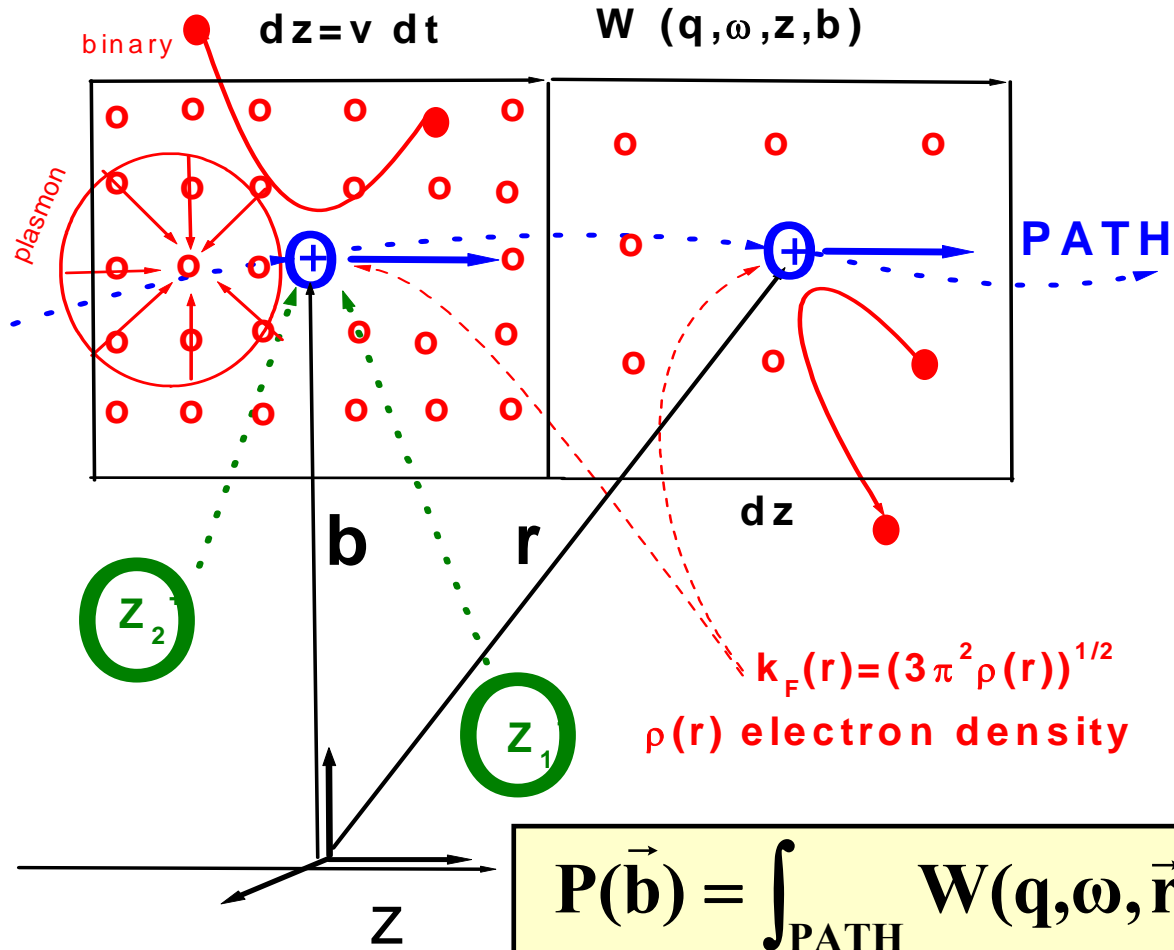
**Straggling (PRA 75, 022903 (2007)).**

**Antiscreening (PRA 67, 062702 (2003) ).**

**Ionization ( PRA 67, 032703 (2003), JPB 36 ,13043 (2003) ).**

**On surfaces (PRA 74, 012902 (2006), 75, 042904 (2007) ).**

# Local Plasma Approximation - LPA



Lindhard *et al* (1953, 67), Brandt-Lundqvist (1966, 1967, 1970),  
 Bonderup (1967), Chu *et al* (1970, 1972)

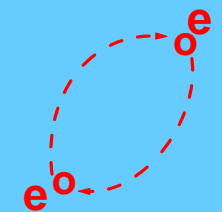
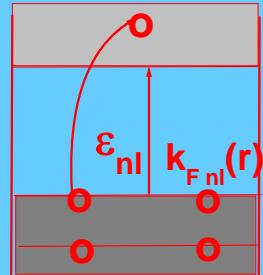
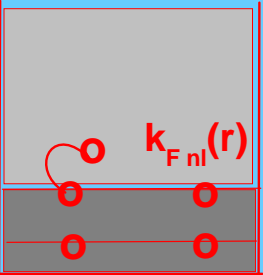
# DIELECTRIC RESPONSE FUNCTION

$$W(q, \omega, \vec{r}) = \frac{2Z_P^2}{\pi v^2} \frac{1}{q} \operatorname{Im} \left[ \frac{1}{\epsilon(q, \omega, k_F(\delta(\vec{r})))} \right]$$

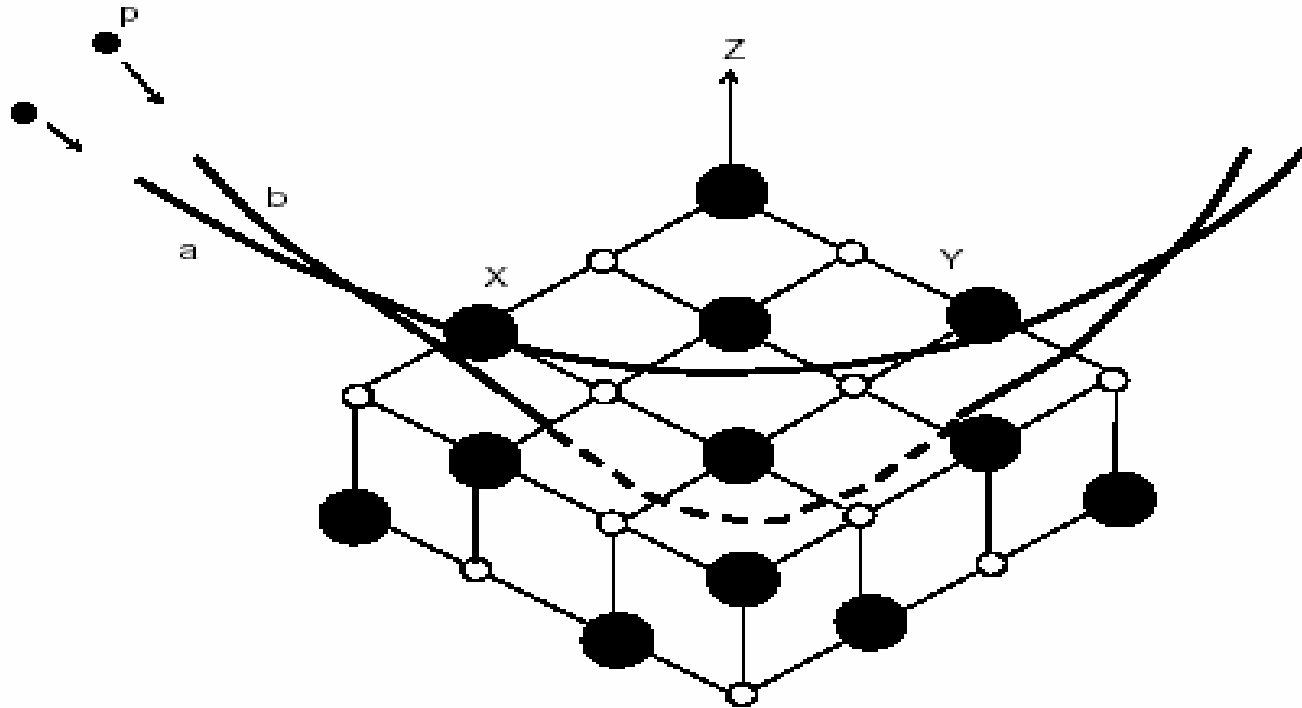
$\epsilon(q, \omega, k_F)$ ,  $k_F = k_F(r)$ , Lindhard (1954),  
it accounts e - e correlation to all orders  
&  $Z_P$  to first order

$\epsilon(q, \omega, k_F, \omega_{nl})$  Levine & Louie (1982)  
↑ to account for the gap & f-sum rule  
shell-to-shell  $\Rightarrow$  independent shell app.

$\epsilon(q, \omega, k_F, \omega_{nl}, S(q))$  Singwi (1968 – 1981)  
↑ to account for e - e exchange & pair  
2 – 3% no more



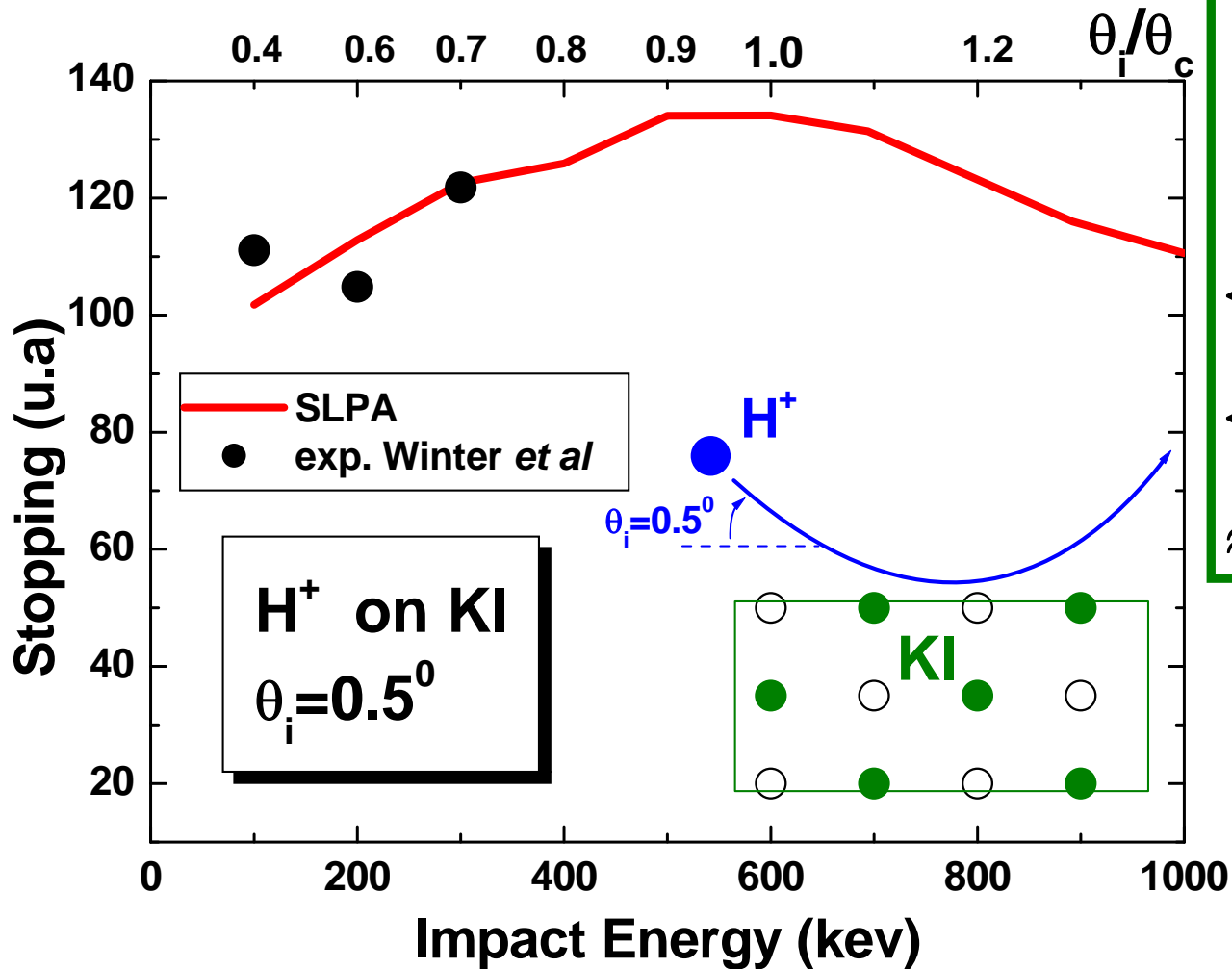
# Collisions with insulator surfaces



16 insulators :

$\text{Li}^+$ ,  $\text{Na}^+$ ,  $\text{K}^+$ ,  $\text{Rb}^+$  with  $\text{F}^-$ ,  $\text{Cl}^-$ ,  $\text{Br}^-$ ,  $\text{I}^-$

García *et al* (2006, 2007), Gravielle *et al* (2007)



$$KI \approx K^+ + I^-$$

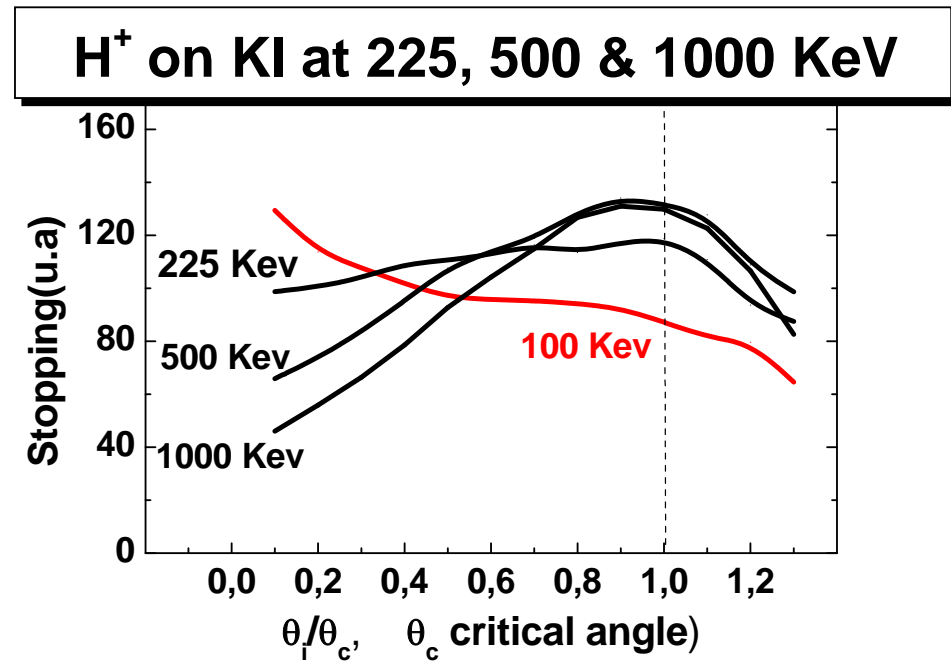
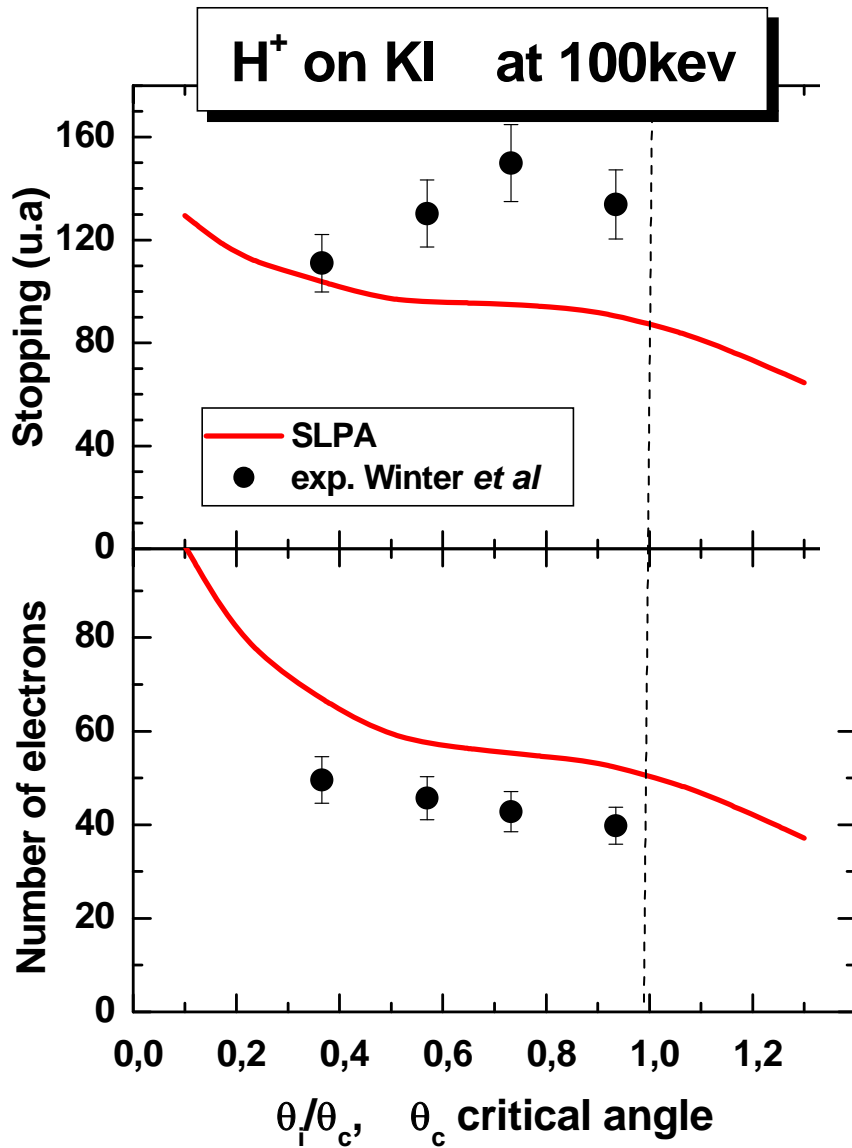
$I^-$  54 elects.

$K^+$  18 elects.

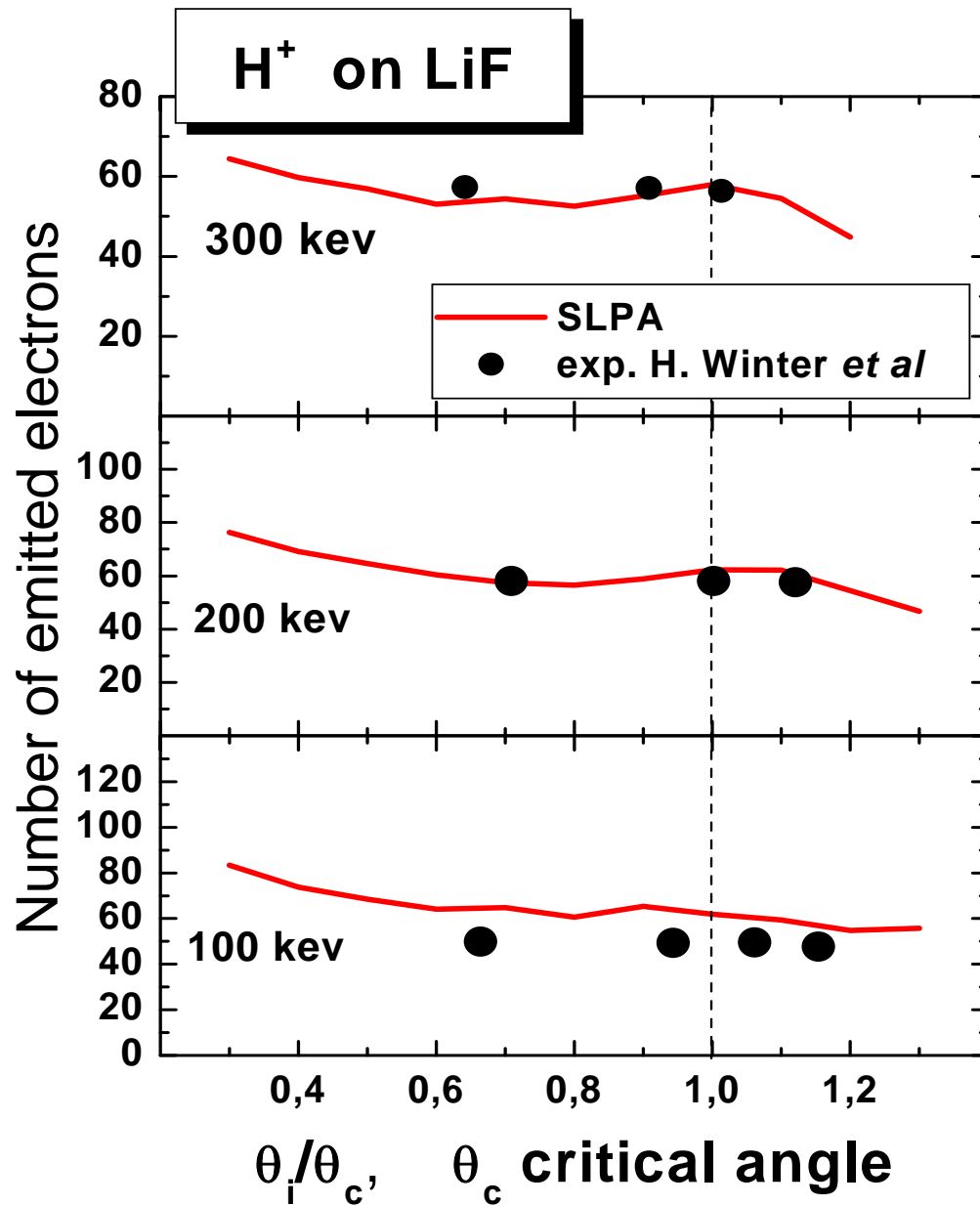
$$\langle r \rangle_{I^- 5p} = 2.7$$

$$\langle r \rangle_{K^+ 3p} = 1.5$$

$\approx 1000$  collisions



E (Kev)	$\theta_c$ (deg.)
100	1.23
225	0.84
500	0.57
1000	0.41



$\text{F}^-$  10 elects.

$\text{Li}^+$  2 elects.

$$\langle r \rangle_{\text{F}^- 2p} = 1.25$$

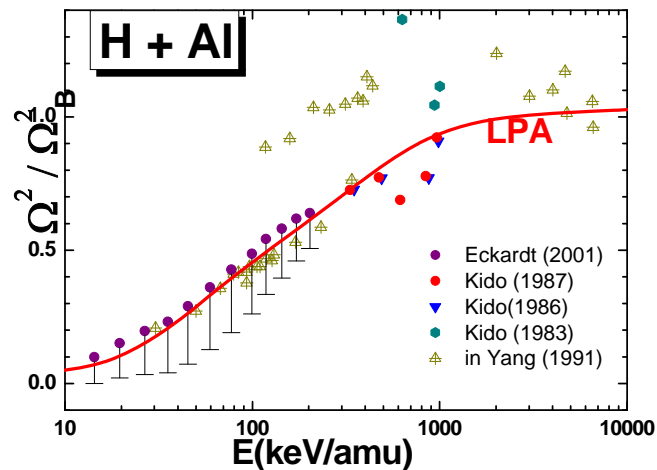
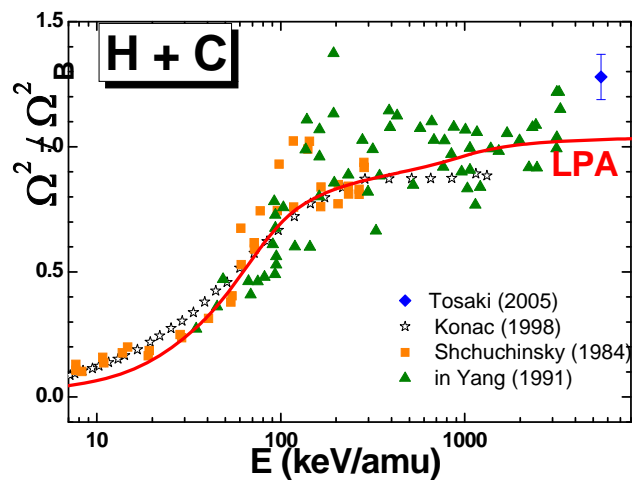
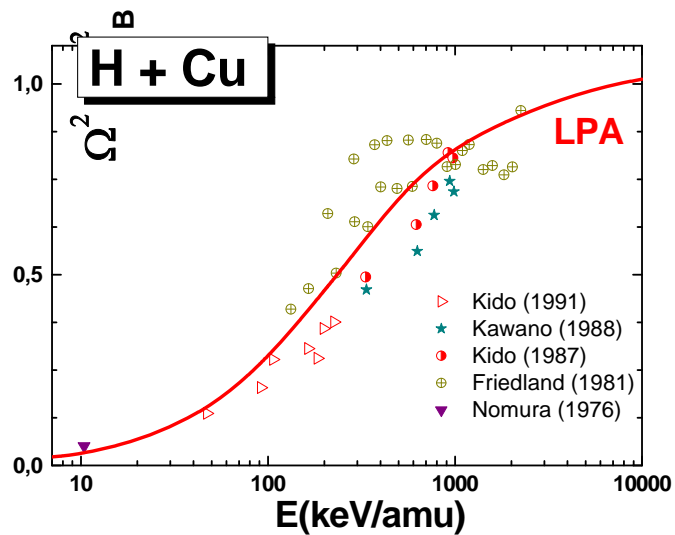
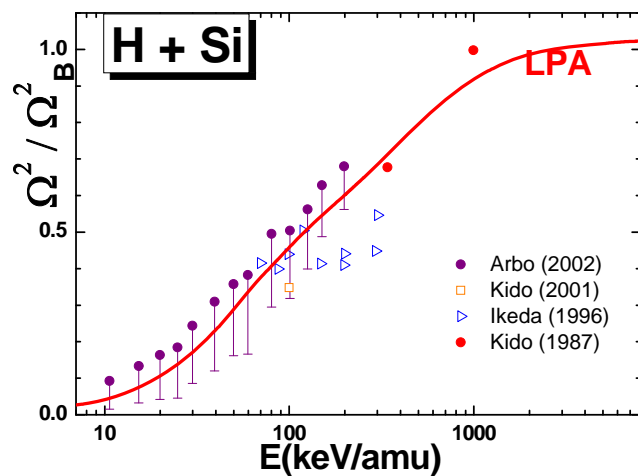
$$\langle r \rangle_{\text{K}^+ 3p} = 0.44.$$

E (Kev)	$\theta_c$ (deg.)
100	0.84
225	0.60
500	0.42
1000	0.31



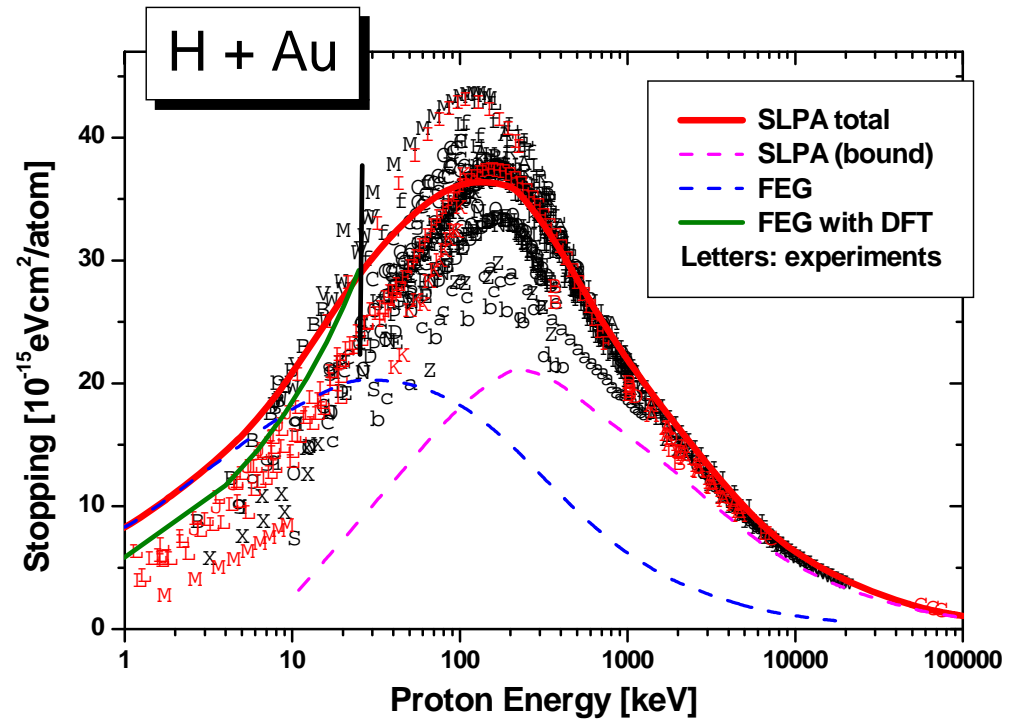
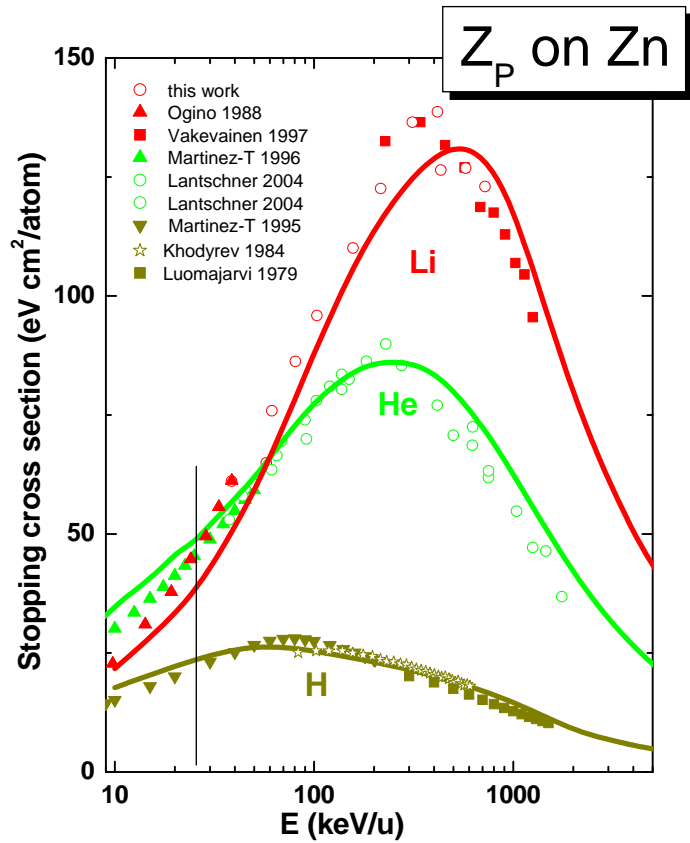
# Straggling

$$\Omega^2 = \int_0^{\infty} d\omega \omega^2 W(\omega), \quad \Omega_B^2 = 4\pi N_e$$



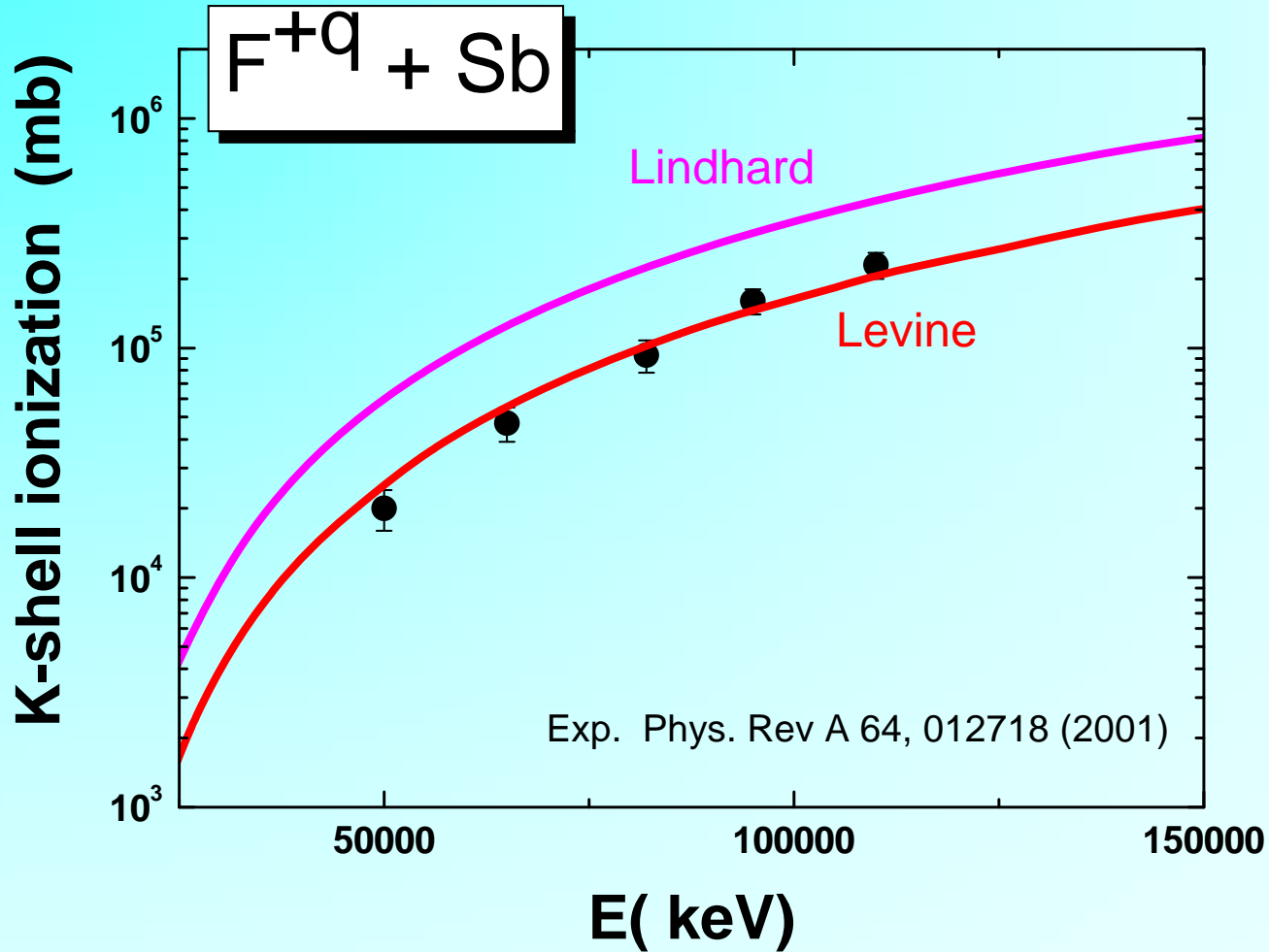
# Stopping

$$S = dE/dl = \int_{\omega_g = \epsilon_{nl}}^{\infty} d\omega \omega^1 W(\omega)$$



# X-section

$$\sigma_{nl} = \int_{\omega_g = \epsilon_{nl}}^{\infty} d\omega \omega^0 W_{nl}(\omega), \text{ / atom}$$



# Multiple Ionization

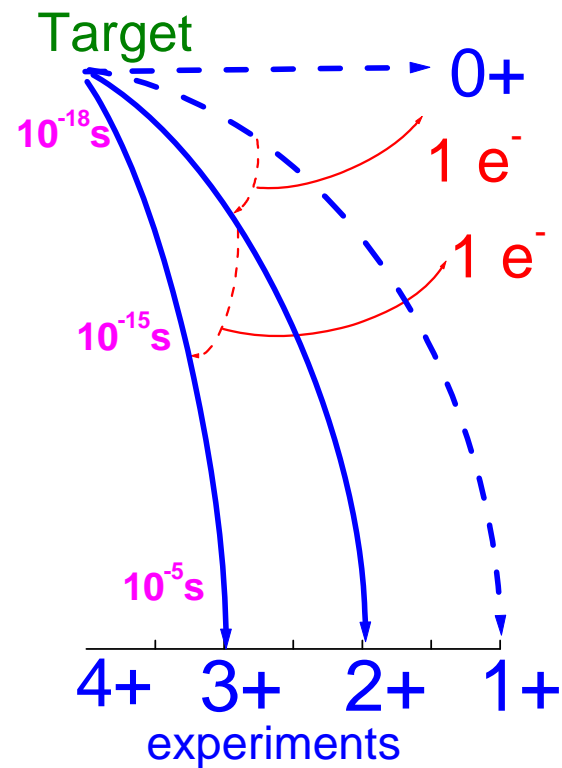
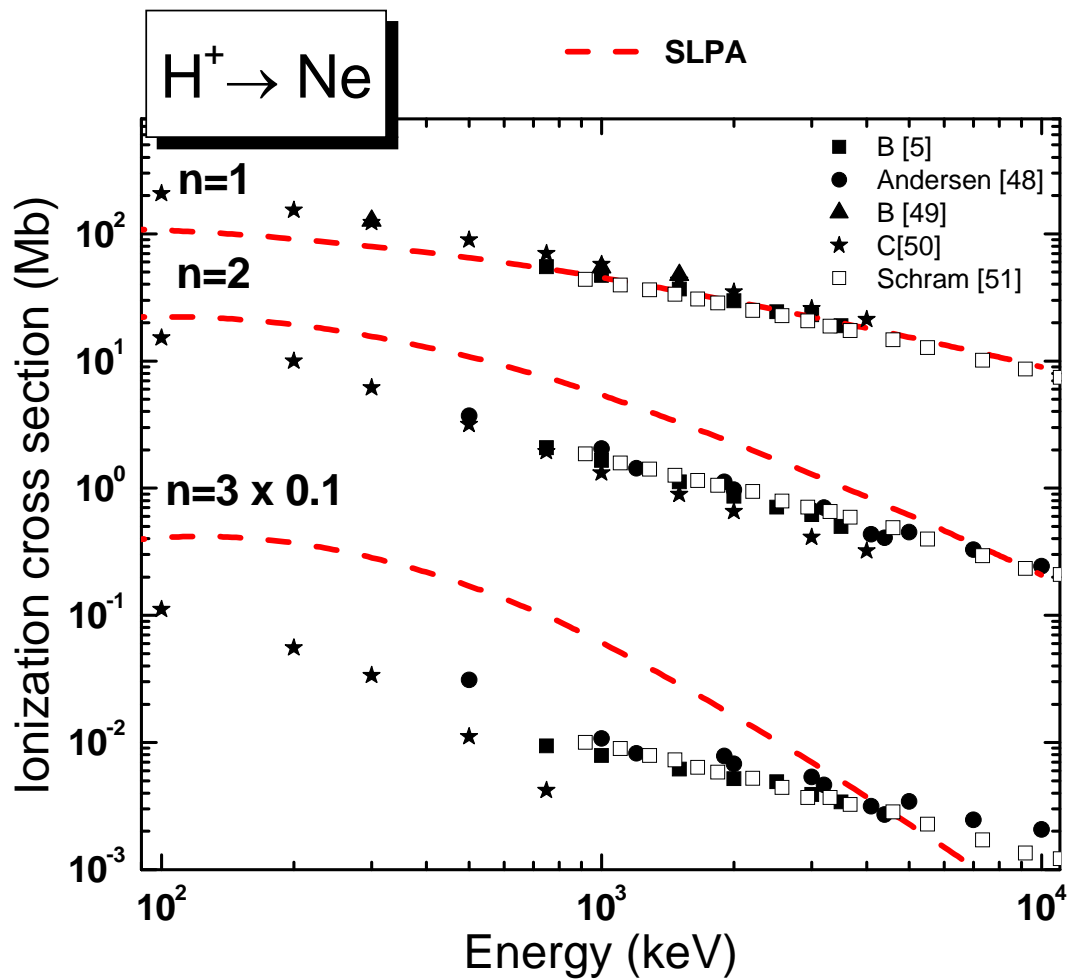
$$P_{nl}(\vec{b}) = \int_{-\infty}^{\infty} dz \int_0^{\infty} d\omega \int_{\omega/v}^{\infty} dq W_{nl}(q, \omega, z, b), \quad nl = \text{shell}$$

$$P(\vec{b}) = \sum_{nl} P_{nl}(\vec{b}) \quad \text{Differential probability}$$

$$P_N(\vec{b}) = \frac{1}{N!} P^N(\vec{b}) \exp[-P(\vec{b})], \quad \text{Poisson dist.}$$

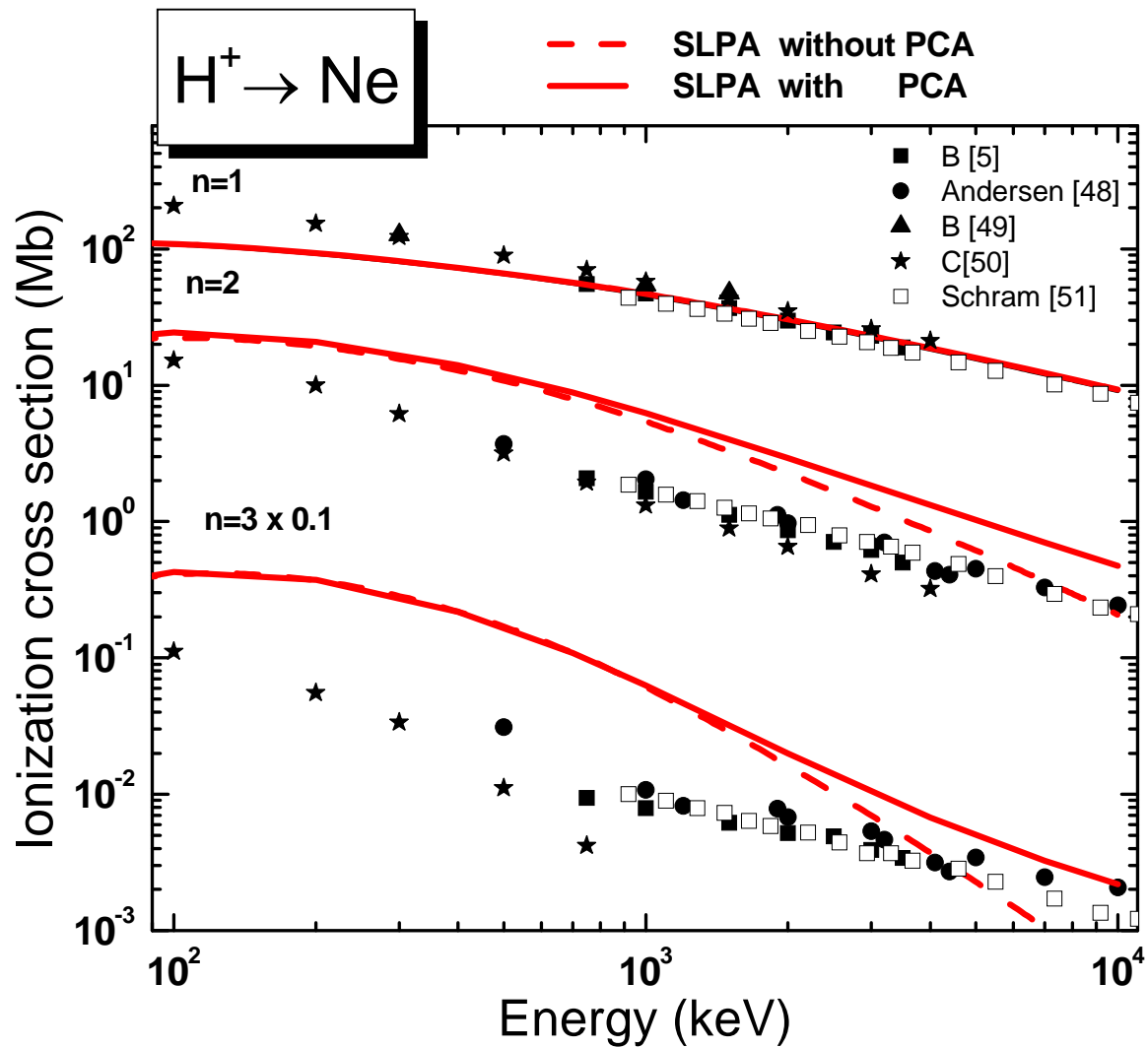
$$\sum_N P_N(\vec{b}) = 1, \quad \text{Normalization}$$

$$\sigma_N = \int d\vec{b} P_N(\vec{b}), \quad \text{Xsection for N-ionization}$$



Experimental ratio

Morgan [21] (1997)

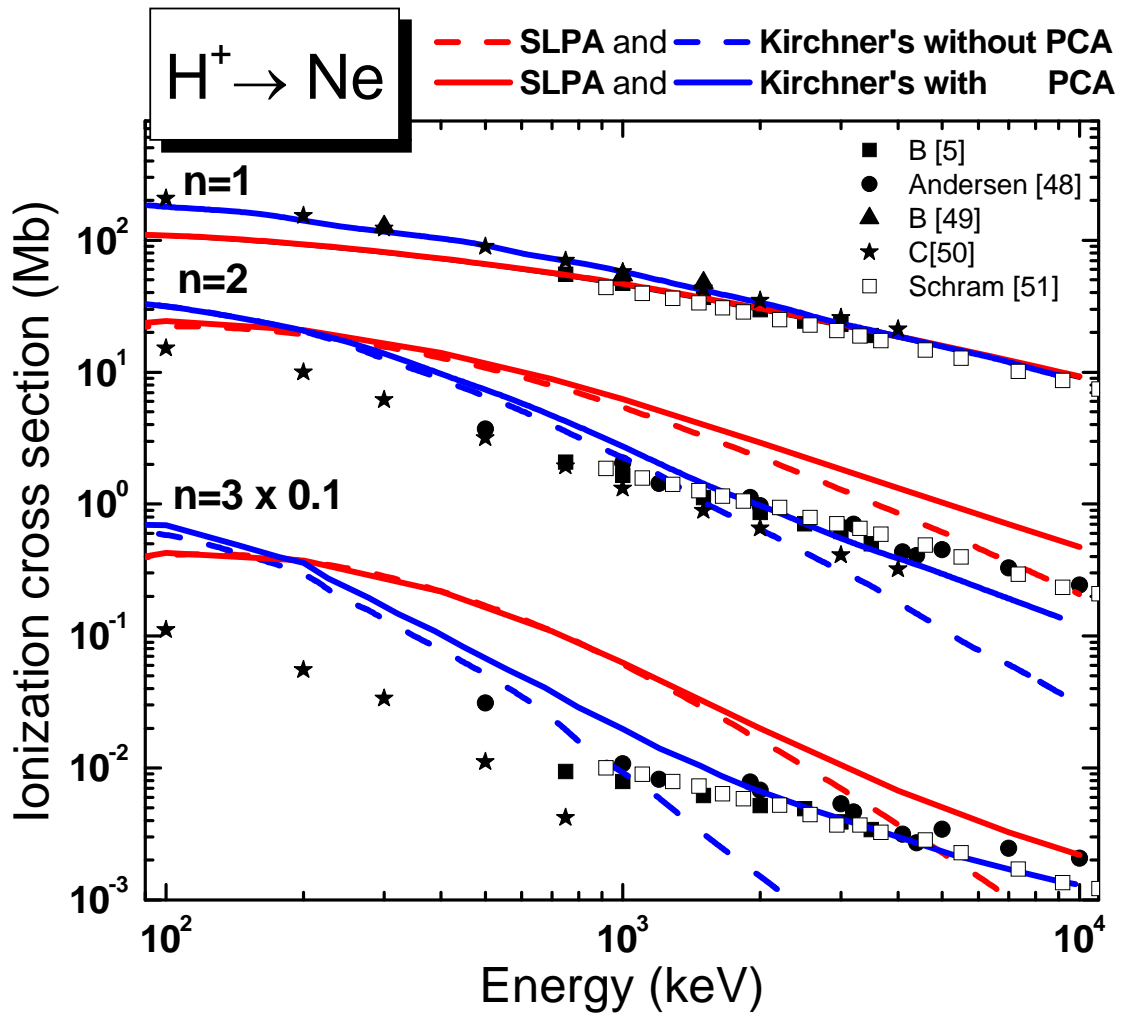


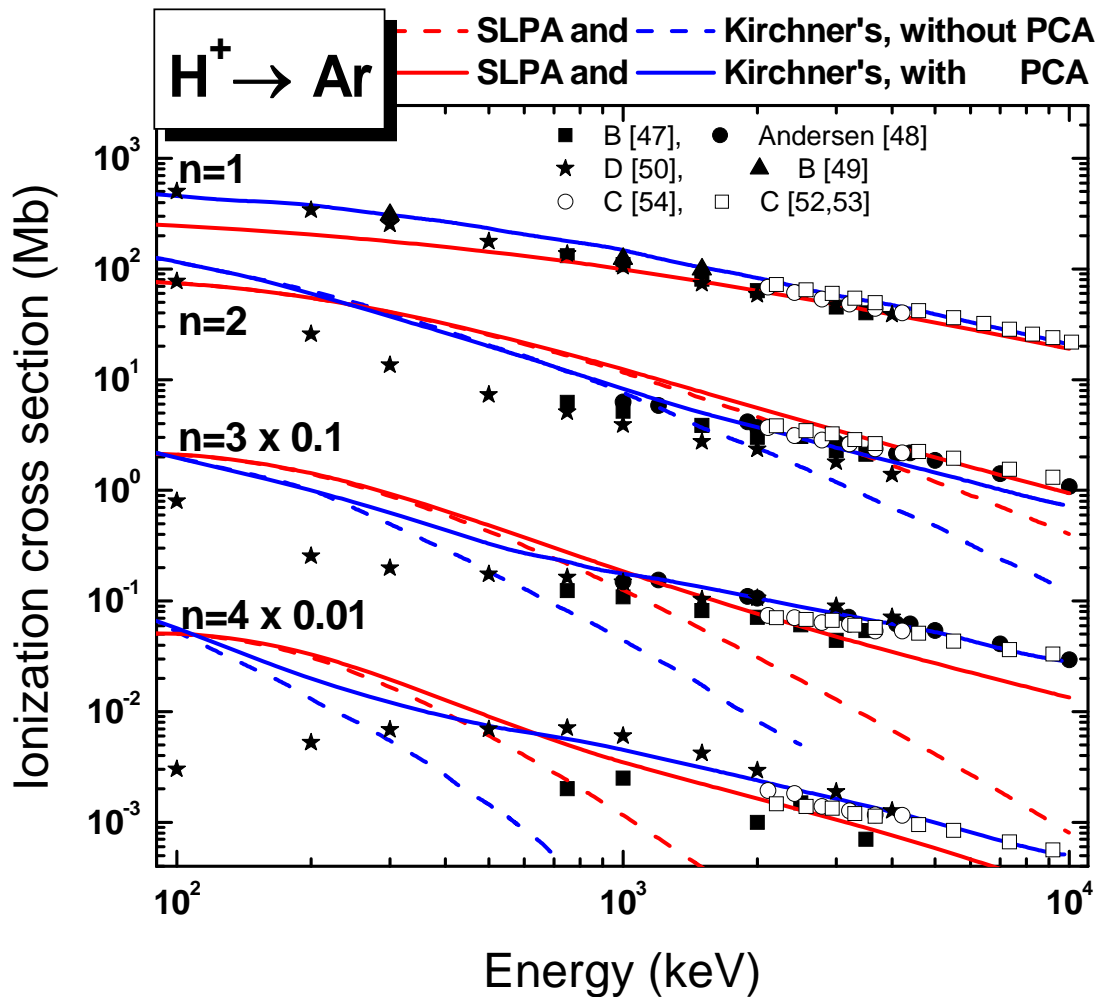
Experimental ratio

Morgan [21] (1997)

Ne	1s
F(nl,1+)	1.5%
F(nl,2+)	93.5
F(nl,3+)	4.8
F(nl,4+)	0.3

Experimental ratios  
Morgan [21] (1997)





### Experimental ratios

Carlson [45] (1966)

Brünken [23] (2002)

**Ar** 1s 2s 2p

F(nl,1+) 0.7 0. 0.5%

F(nl,2+) 10.5 1. 86.2

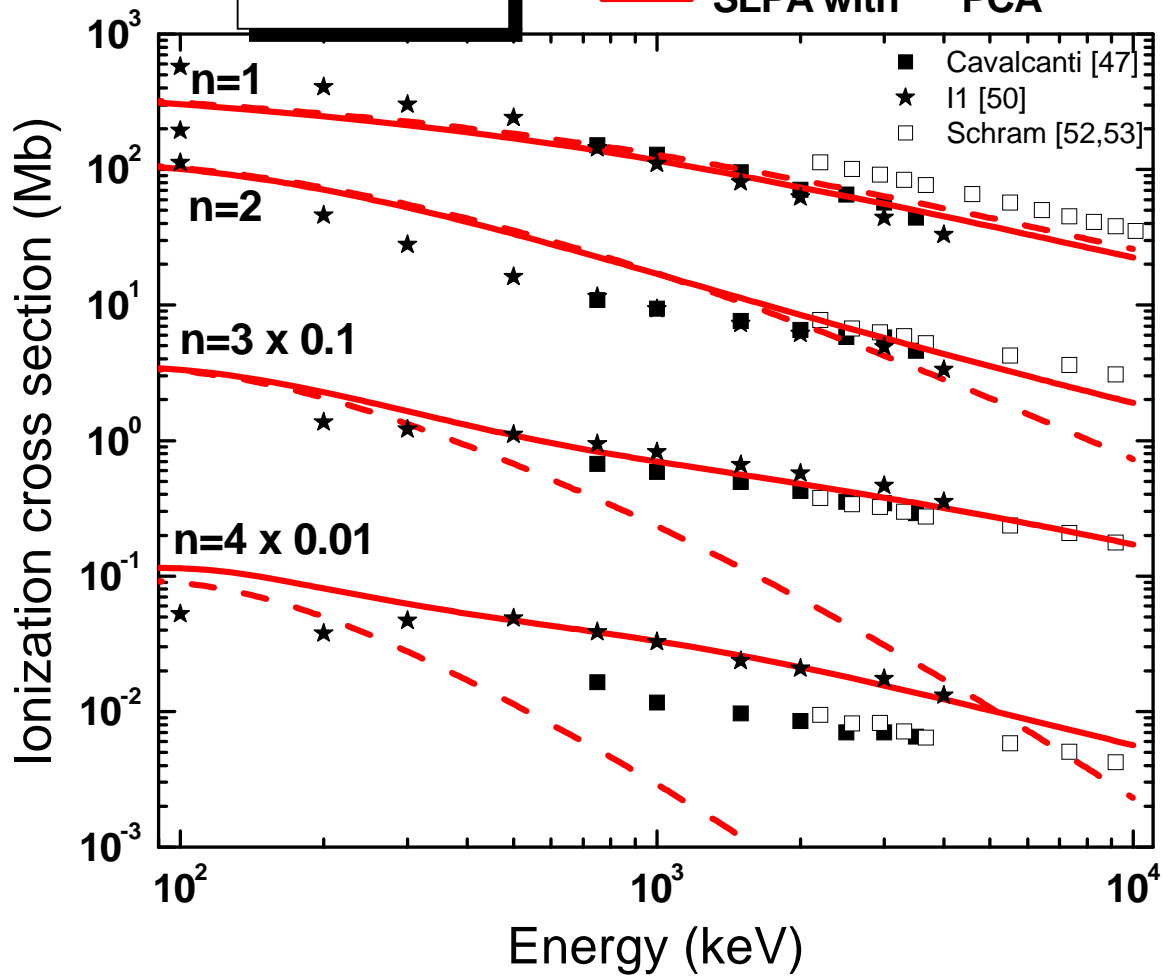
F(nl,3+) 7.8 89. 13.0

F(nl,4+) 42.7 10. 0.3



**H<sup>+</sup> → Kr**

--- SLPA without PCA  
 --- SLPA with PCA



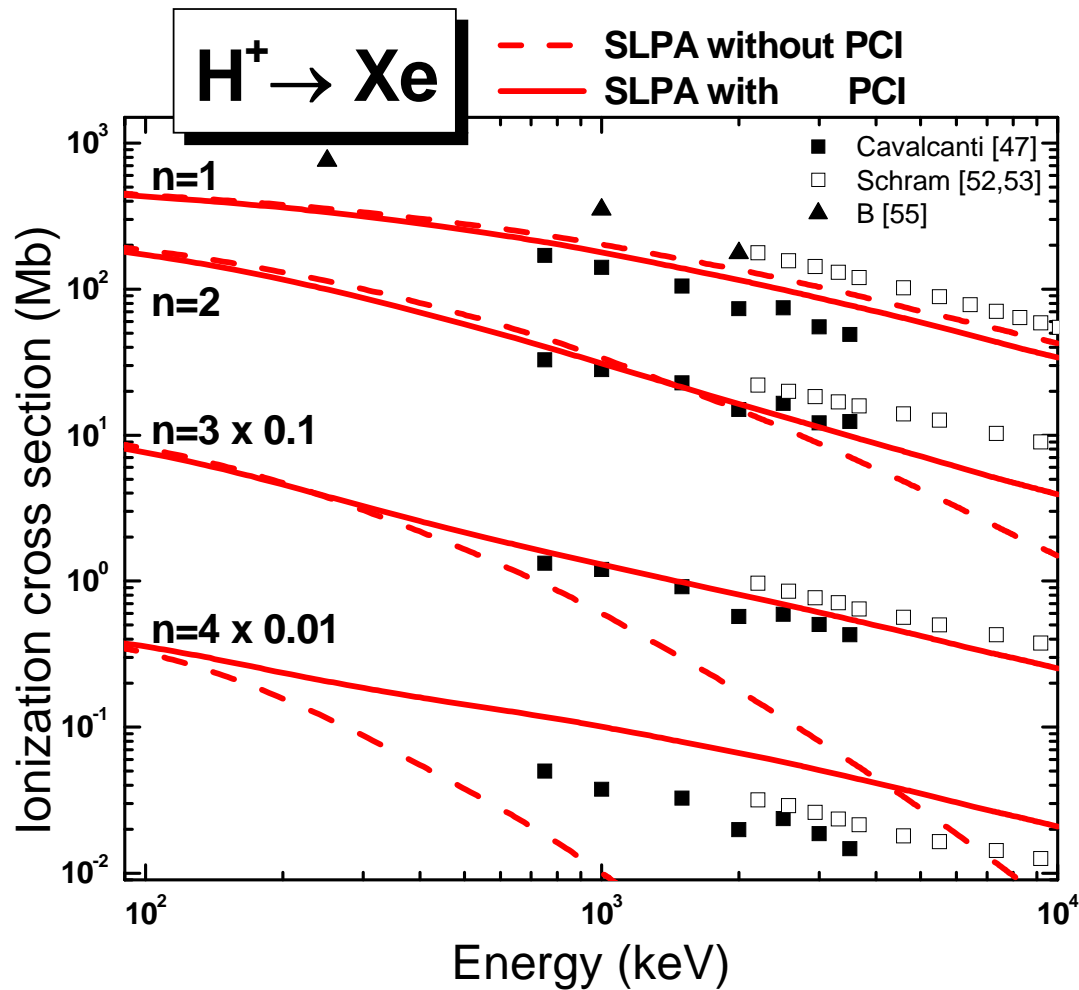
Experimental ratios

Carlson [45] (1966)

Armen [22] (2004)

Tamenori [24] (2004)

Kr	1s	2s	2p	3s	3p	3d
F(nl,1+)	1.	0.3	0.	0.	0.	2.%
F(nl,2+)	1.5	0.1	0.8	13.	3.5	56.
F(nl,3+)	6.	0.3	2.3	42.	58.6	38.
F(nl,4+)	17.	1.	31.	30.	32.6	3.



### Experimental ratios

Carlson [45] (1966)

Saito [26] (1992)

Tamenori [27] (2004)

Hayaishi [28] (2002)

Kämmerling [29] (1992)

<b>Xe</b>	3s	3p	3d	4p	4d	N-shell
F(nl,1+)	0	0	0	0	1	5%
F(nl,2+)	0	0	0.3	0	80	40
F(nl,3+)	0	0	4	66	19	28
F(nl,4+)	0	0	39	34	0	21

# Resumé

## Advantages of the SLPA:

1- e-e correlation to all order

2- Just the electron densities & binding energies.

Do not need the continuum. Good for DFT used in QCh.

3- Cartesian coordinates. Not needed central potential

4- Projectile classical trajectory selfconsistent (e impact)

## Disadvantages

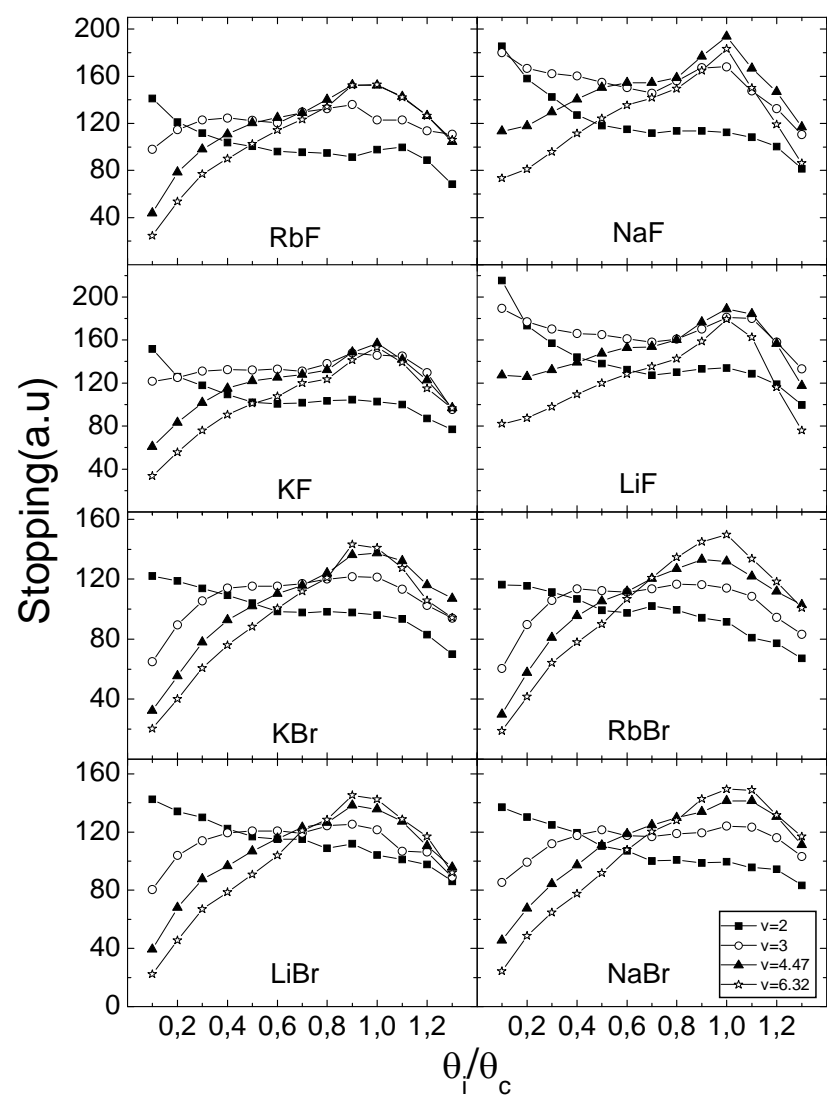
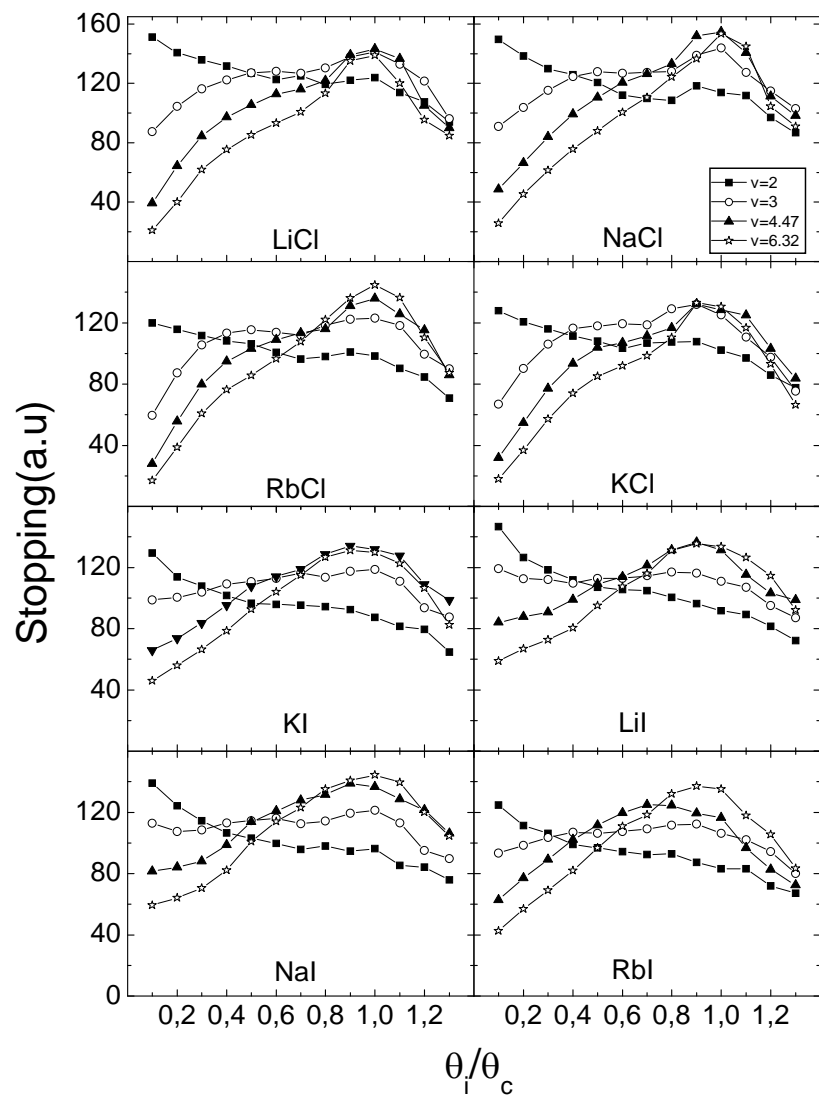
1- First order in the projectile charge

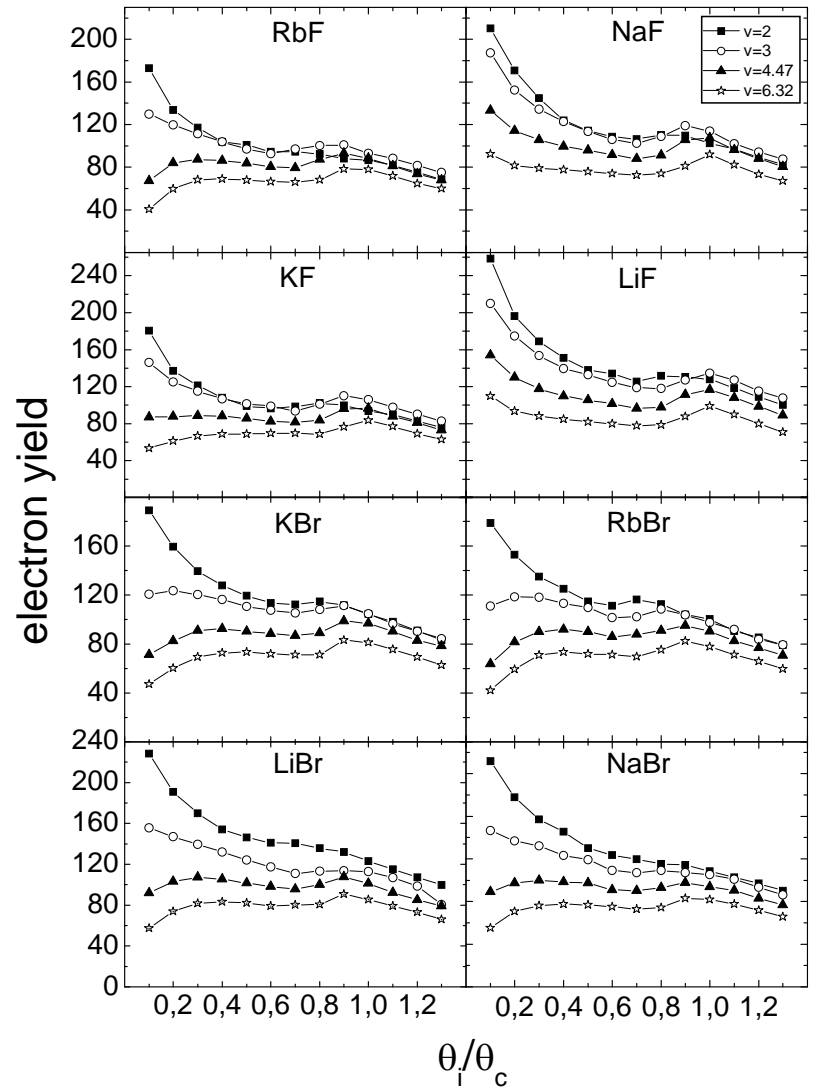
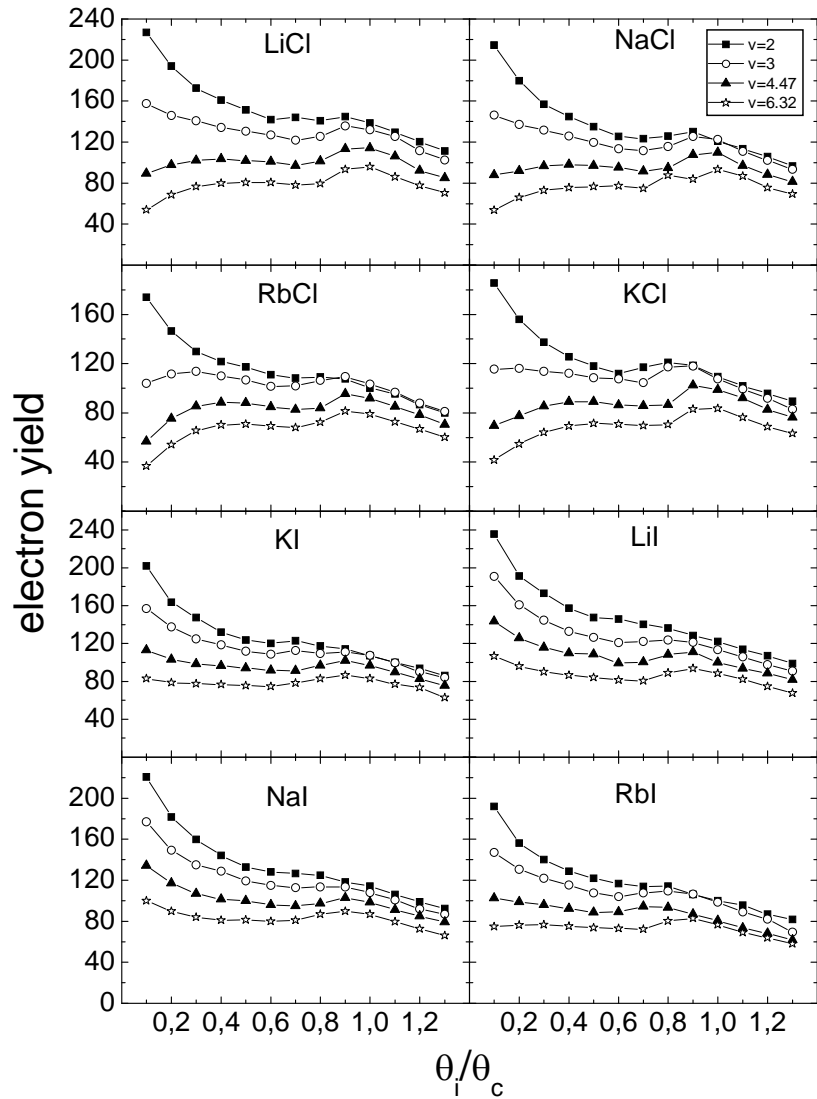
2- It is local

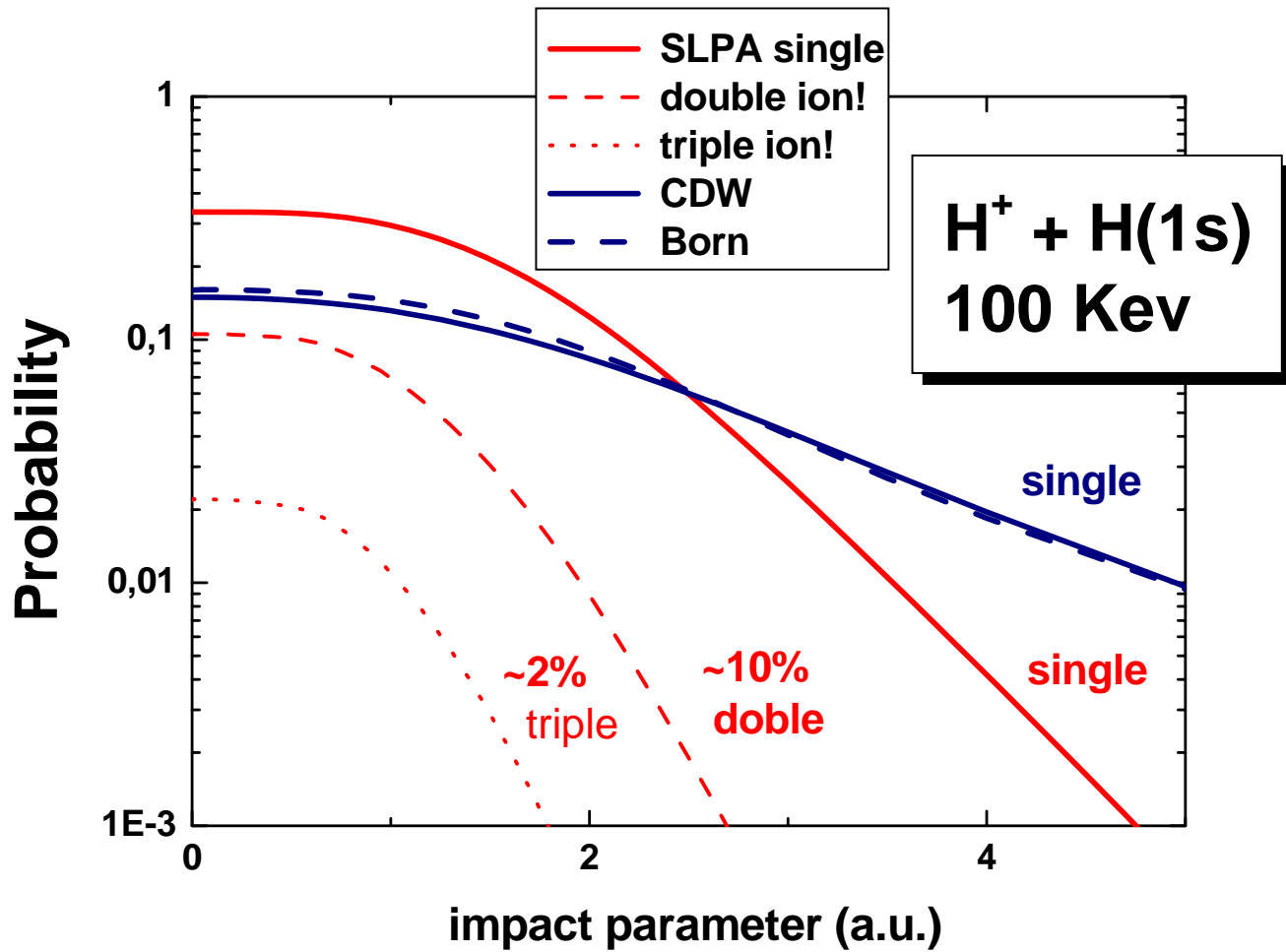
3- It is a model. No perturbative series to follow

## **Future Developments**

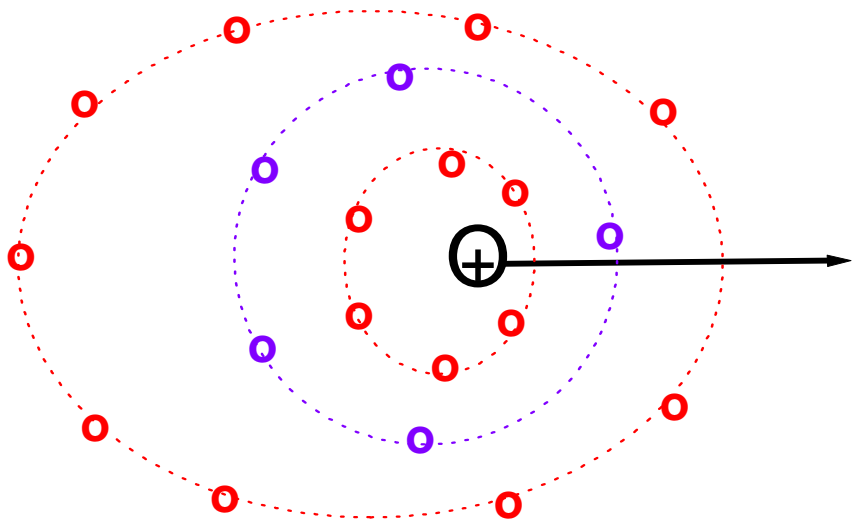
- 1- Heavy atoms f-shell , molecules & clusters**
- 2- Atom-atom antiscreening (= collision of two FEG)**
- 3- Improve the Local hypothesis by extending to momentum space. Intense activity in QCh**





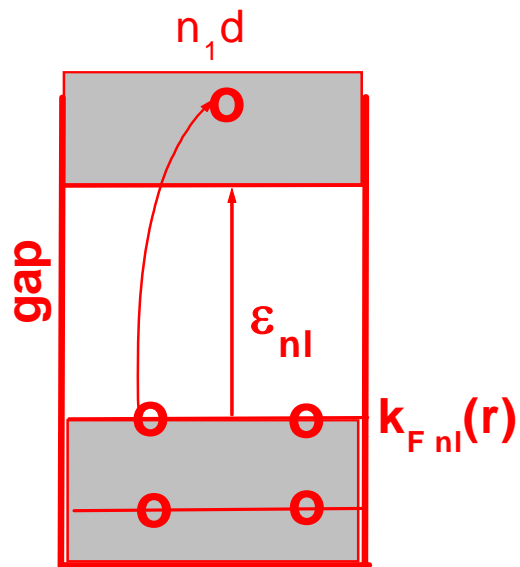
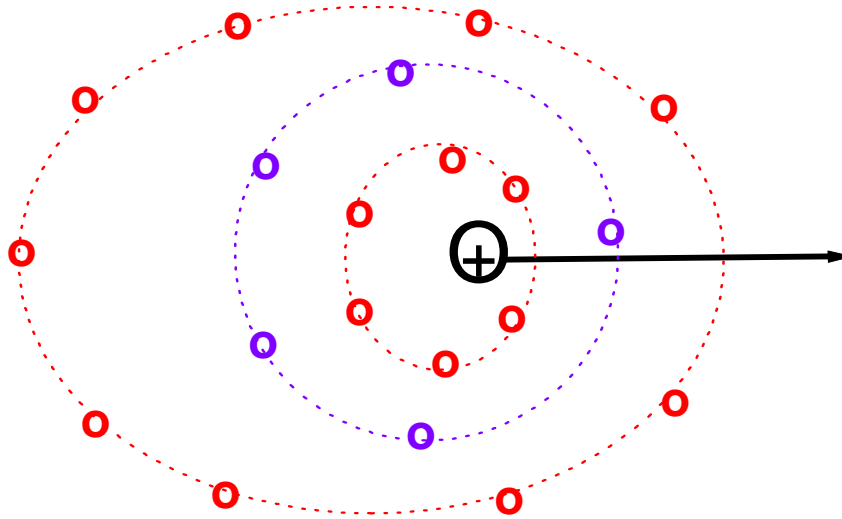


# Independent Shell approximation - SLPA

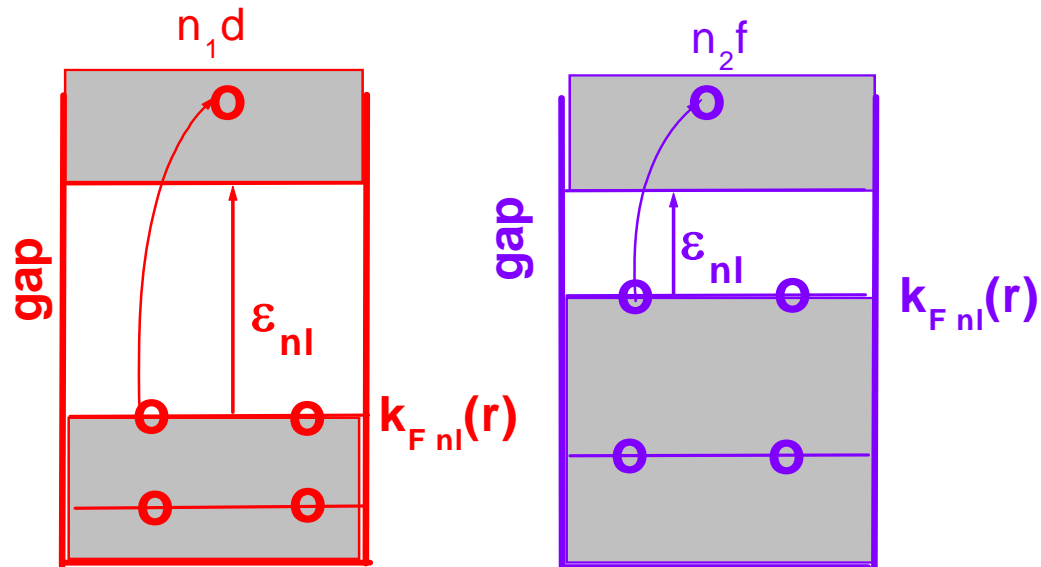
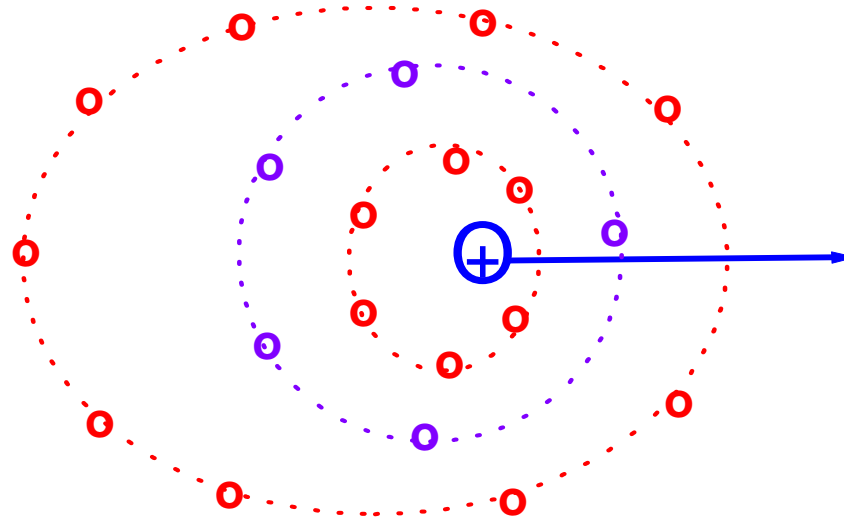




# Independent Shell approximation - SLPA



# Independent Shell approximation - SLPA



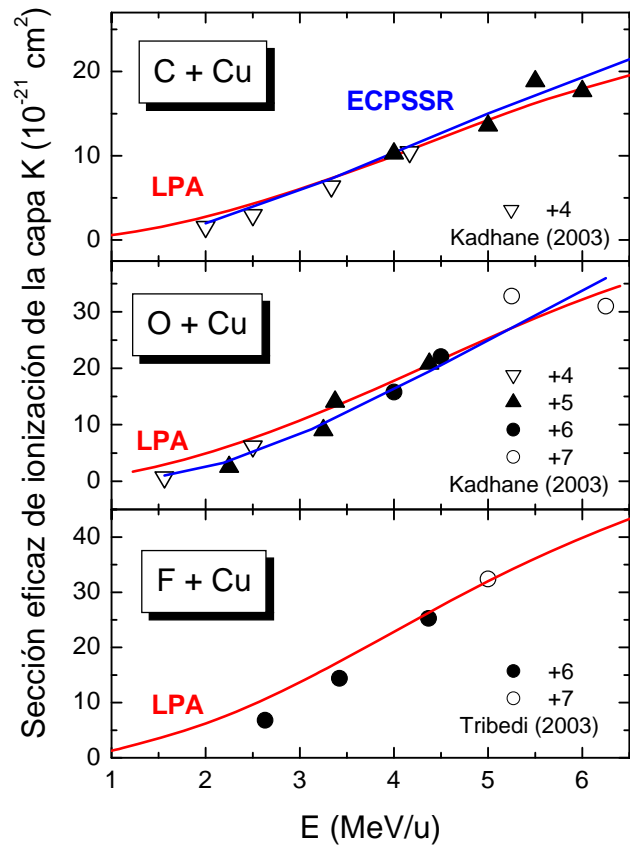
## Boltzman (transport) Equation

$$\frac{\partial g(E, z, b)}{\partial z} = \int_0^{E_0} d\omega \ W(\omega, z, b) [g(E + \omega, z, b) - g(E, z, b)]$$

$g(E, z, b) =$  **Projectile energy distribution function**

$$W(\omega, z, b) = \mathbf{Kernel} = \int_{\omega/v}^{\infty} dq \ W(\omega, q, z, b)$$

$$W(\omega, q, z, b) = \frac{2Z_P^2}{\pi v^2} \frac{1}{q} \operatorname{Im} \left[ \frac{1}{\varepsilon(q, \omega, k_F(\vec{r}))} \right]$$



charged ions in photoionization experiments. This ratios  $F_{nl,i}$  express how the direct single ionization of a certain  $nl$  shell contributes to ion formation of charge  $+i$ , so that  $\sum_i F_{nl,i} = 1$ . Including these ratios in (14) we obtain

$$p(b) = \sum_{nl} p_{nl}(b) \times 1 = \sum_{nl} p_{nl}(b) \times \sum_i F_{nl,i} = \sum_{i=1}^{Z_T} \mathcal{P}_i(b) \quad (16)$$

with

$$\mathcal{P}_i(b) = \sum_{nl} p_{nl}(b) F_{nl,i} \quad (17)$$

being the probability of a direct single ionization (of any shell  $nl$ ) followed by post-collisional emission of  $i - 1$  electrons. Replacing (16) in (10) and rearranging terms so

as to put together those that contribute to the same number of final emitted electrons, we can express ionization probabilities including post-collisional emission as  $P_N^{post}(b)$

$$P_1^{post} = e^{-p(b)} \mathcal{P}_1, \quad (18a)$$

$$P_2^{post} = e^{-p(b)} \left[ \mathcal{P}_2 + \frac{\mathcal{P}_1^2}{2!} \right], \quad (18b)$$

$$P_3^{post} = e^{-p(b)} \left[ \mathcal{P}_3 + \frac{2\mathcal{P}_1\mathcal{P}_2}{2!} + \frac{\mathcal{P}_1^3}{3!} \right], \quad (18c)$$

$$P_4^{post} = e^{-p(b)} \left[ \mathcal{P}_4 + \frac{2\mathcal{P}_1\mathcal{P}_3}{2!} + \frac{\mathcal{P}_2^2}{2!} + \frac{3\mathcal{P}_1^2\mathcal{P}_2}{3!} + \frac{\mathcal{P}_1^4}{4!} \right], \quad (18d)$$

where we have shortened  $\mathcal{P}_N(b)$  and  $P_N^{post}(b)$  by  $\mathcal{P}_N$  and  $P_N^{post}$ . Each expression  $P_N^{post}$  includes the term  $e^{-p(b)} \mathcal{P}_1^N / N!$  that considers all the combination of direct ionization probabilities of  $N$  electrons of different shells, and the term  $e^{-p(b)} \mathcal{P}_N$  that corresponds to a single ionization followed by the emission of  $N - 1$  electrons via Auger