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# Doubly Inelastic Collisions with Relativistic Heavy Ions and Target Recoil Momentum Spectroscopy

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XX ISIAC August 1-4 2007, Agios Nikolaos, Crete, GREECE

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# Outline

- **Introduction**
  - **Theoretical approaches**
    - **First-Born Approximation: FBA**
    - **Eikonal Model: EA**
  - **Results**
  - **Conclusions**
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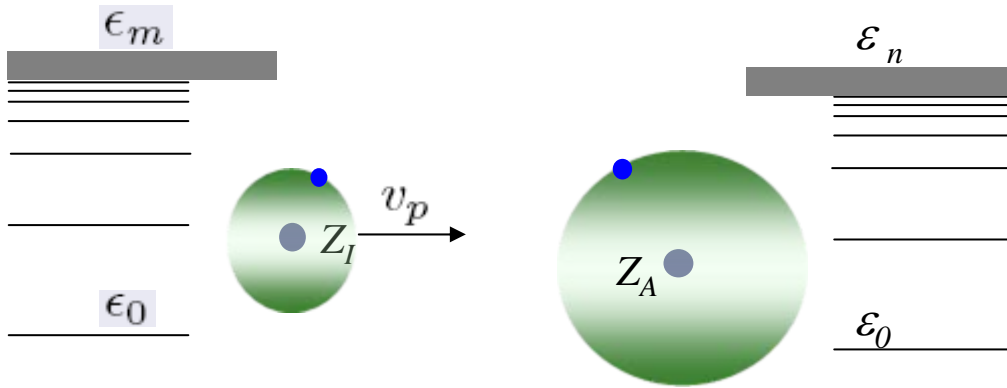
# Introduction

- $v_p \ll c$ , two point-like charges interact via the Coulomb law.

Despite this law has a very simple form, it is **difficult** to get detailed understanding of the dynamics of atomic collisions involving **more than two** active particles;

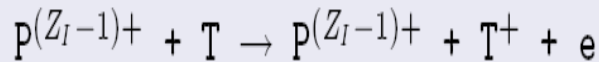
- $v_p \sim c$ , the form of the pair-wise EM interaction becomes more complicated.
    - Detailed description of relativistic collisions involving more than two active particles, represents a particularly **strong challenge for theory**.
    - Phenomena occurring in such collisions are **less understood compared to their nonrelativistic counterparts**.
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# Introduction



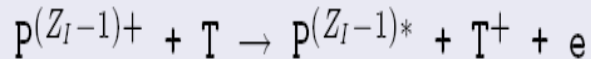
## Singly inelastic collisions

- **Target ionization**

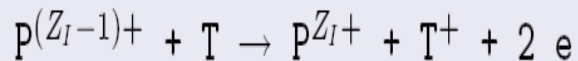


## Doubly inelastic collisions

- **Mutual P-excitation and T-ionization**



- **Mutual P-loss and T-ionization**



## Energy-momentum conservation

$$\mathbf{q}_A = (\mathbf{Q}, q_{min}^A)$$

$$q_{min}^A = \frac{\epsilon_n - \epsilon_0}{v_p} + \frac{\epsilon_m - \epsilon_0}{\gamma v_p},$$

$$\mathbf{q}_I = (-\mathbf{Q}, -q_{min}^I)$$

$$q_{min}^I = \frac{\epsilon_n - \epsilon_0}{\gamma v_p} + \frac{\epsilon_m - \epsilon_0}{v_p},$$

$v_p$  : collision velocity,

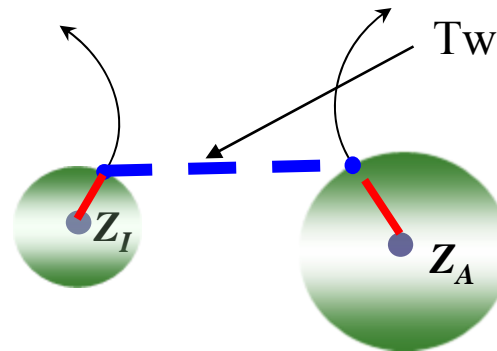
$\gamma$  : collision Lorentz factor.

$$q_{min}^A = p_{R,||} + k_{||}$$

# First-Born approximation

A.B.V. **PR 392**, 191 (2004)

$$Z_I / v_p \ll 1$$

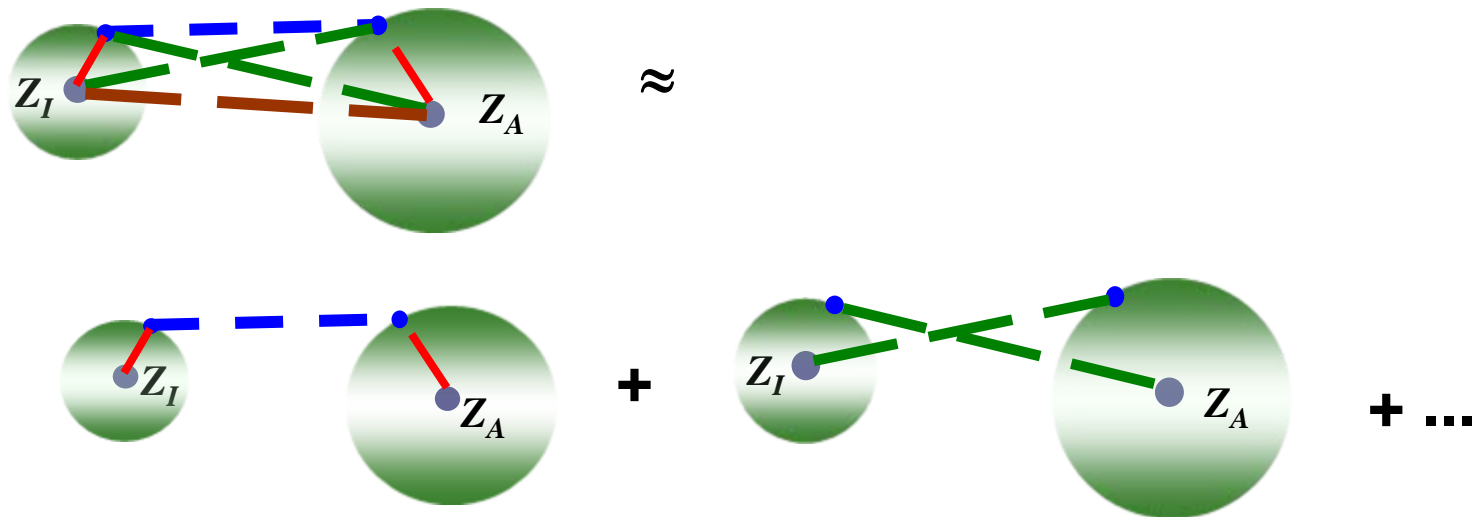


Two-Center Dielectronic Interaction

A.B.V. et al **PRL 92**,  
213202 (2004)

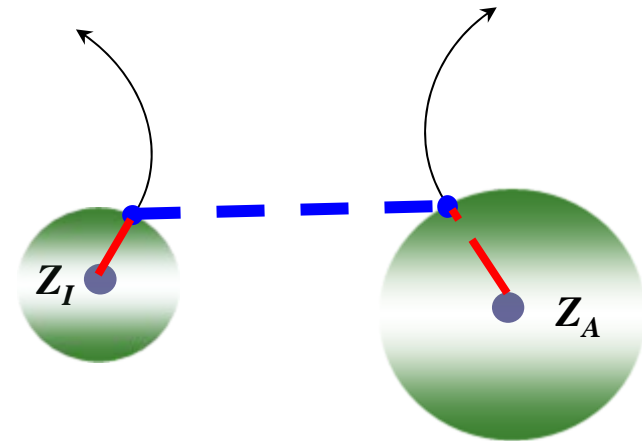
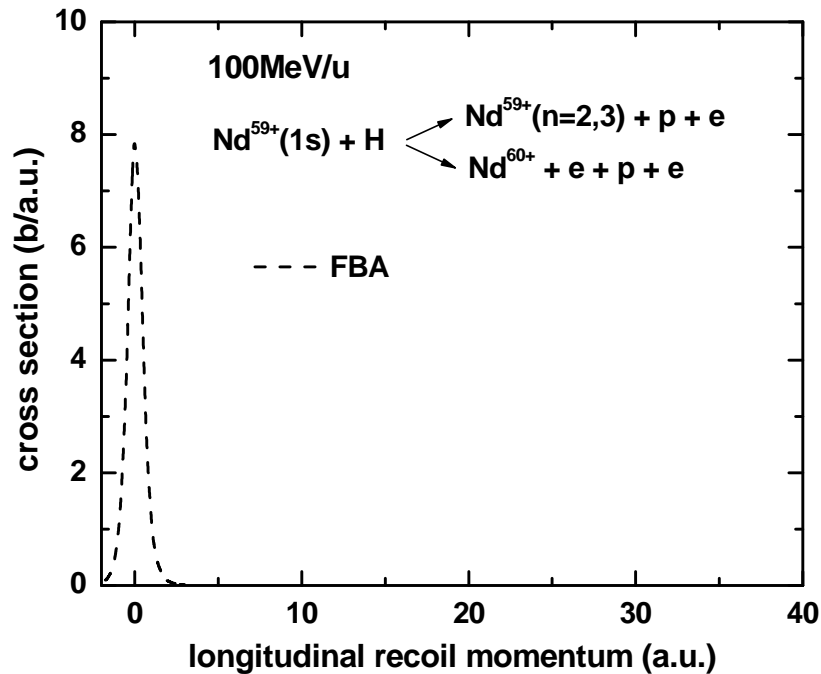
## Eikonal model

A.B.V. and B.N. **JPB 38**, 3587 and  
A.B.V. **PRA 72**, 062705 (2005)



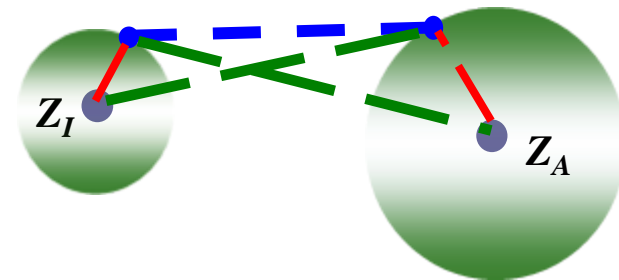
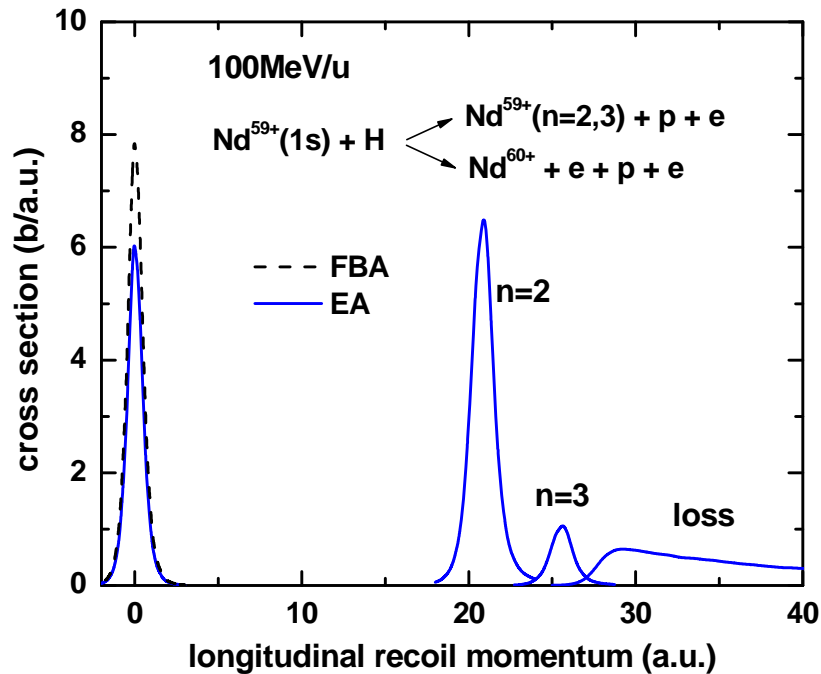
# Results: Target-recoil momentum spectra

## Doubly inelastic collisions



# Results: Target-recoil momentum spectra

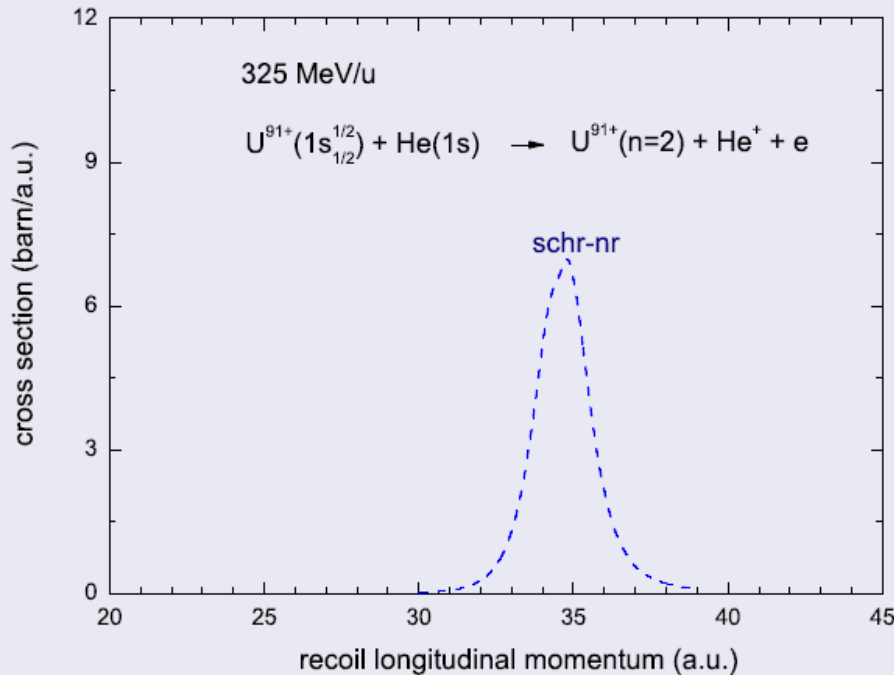
## Doubly inelastic collisions



The peaks at large  $p_{R,\parallel}$  represent clear signatures of the higher order effects.

# Results: Target-recoil momentum spectra

## Doubly inelastic collisions



$$c \rightarrow \infty$$

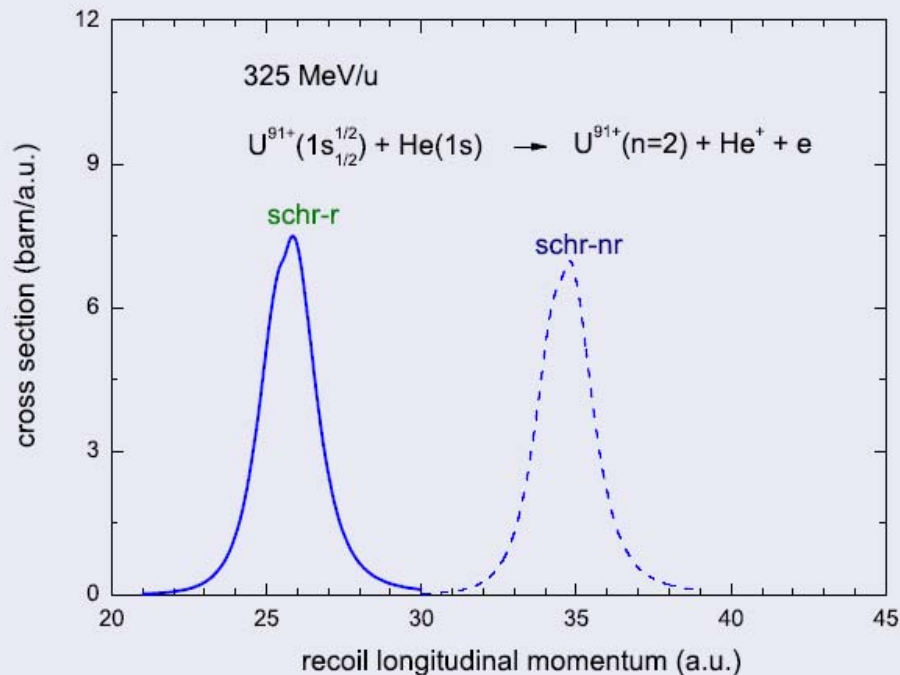
- in the treatment of the **relative ion-atom motion**;
- in the description of the **internal electron states**.

$$q_{min}^A = \frac{\epsilon'_n - \epsilon'_0}{v_p} + \frac{\epsilon'_m - \epsilon'_0}{v_p}$$



# Results: Target-recoil momentum spectra

## Doubly inelastic collisions



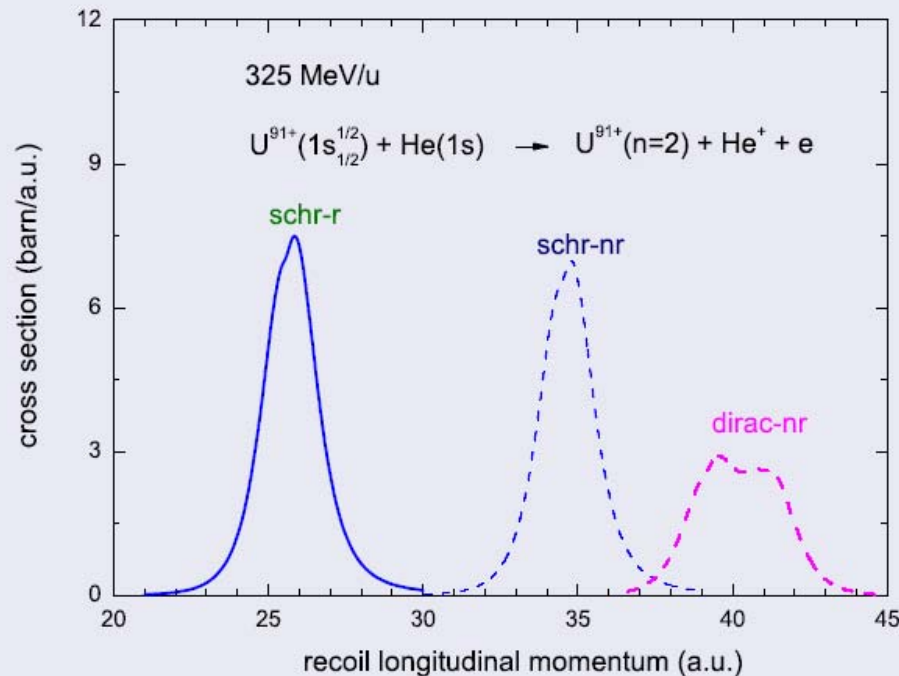
$$c \rightarrow \infty$$

only in the description of the **internal** electron states.

$$q_{min}^A = \frac{\epsilon'_n - \epsilon'_0}{v_p} + \frac{\epsilon'_m - \epsilon'_0}{\gamma v_p}$$

# Results: Target-recoil momentum spectra

## Doubly inelastic collisions



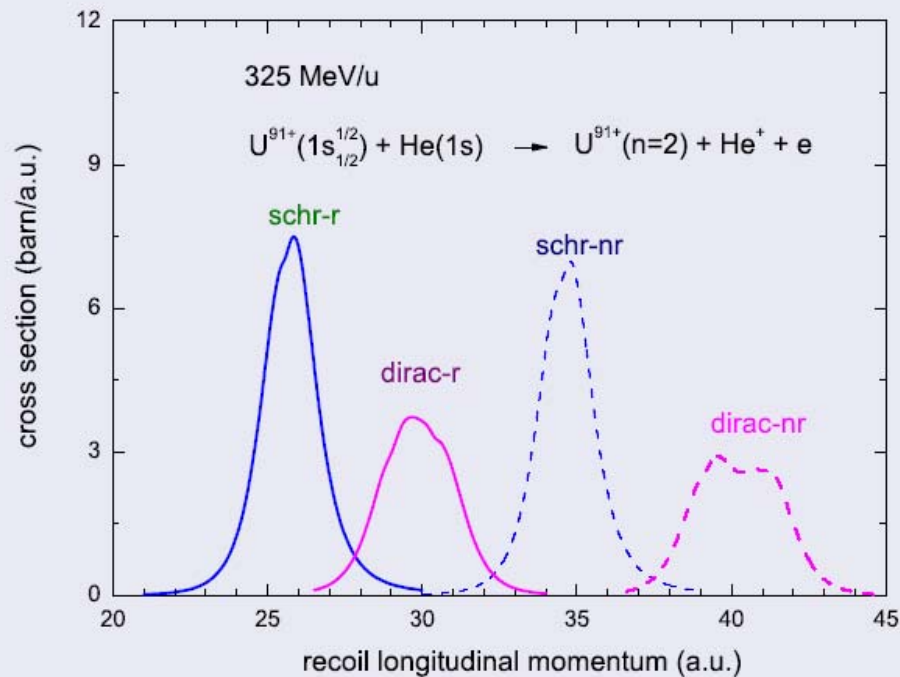
$$c \rightarrow \infty$$

only in the treatment of the relative ion-atom motion.

$$\gamma \rightarrow 1$$
$$q_{min}^A = \frac{\epsilon_n - \epsilon_0}{v_p} + \frac{\epsilon_m - \epsilon_0}{v_p}$$

# Results: Target-recoil momentum spectra

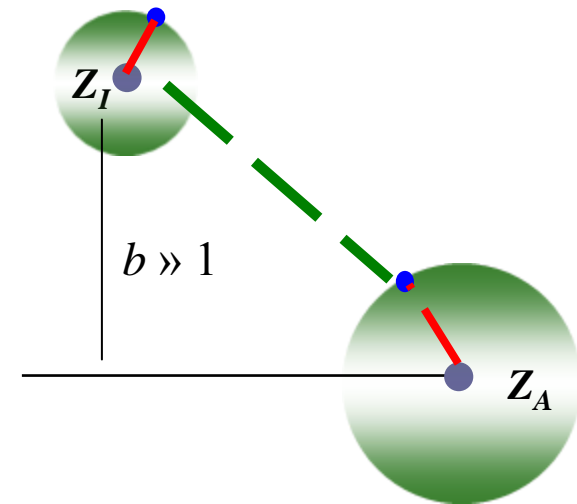
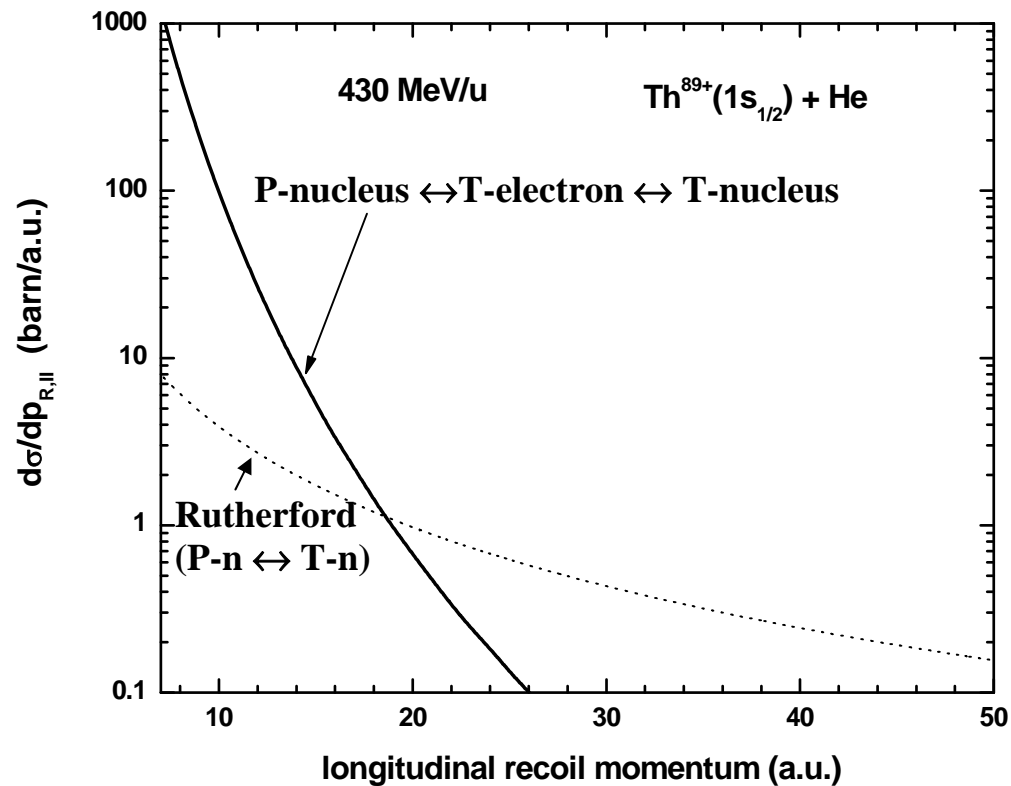
## Doubly inelastic collisions



$$q_{min}^A = \frac{\epsilon_n - \epsilon_0}{v_p} + \frac{\epsilon_m - \epsilon_0}{\gamma v_p}$$

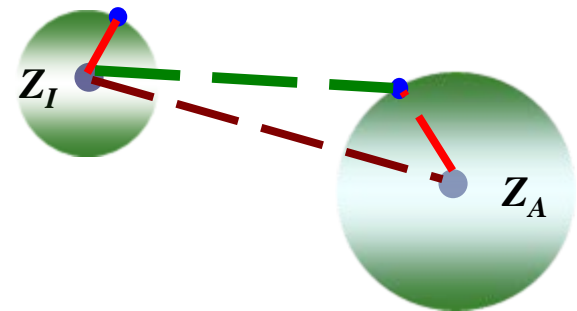
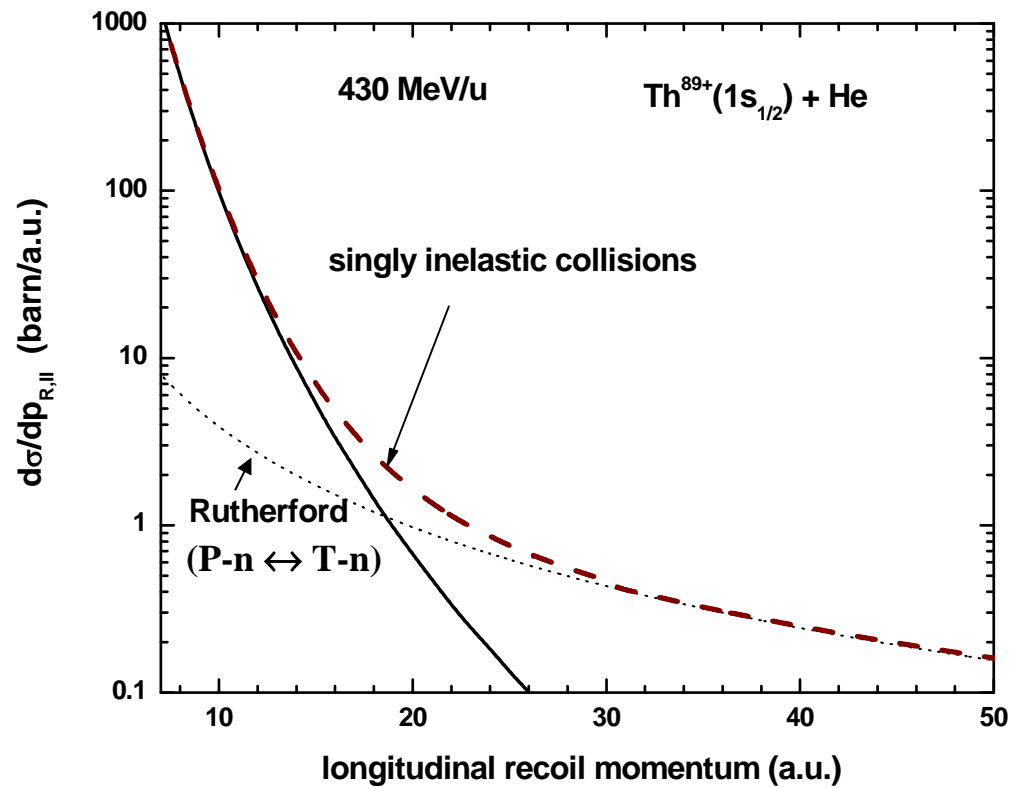
# Results: Target-recoil momentum spectra

## Singly inelastic collisions



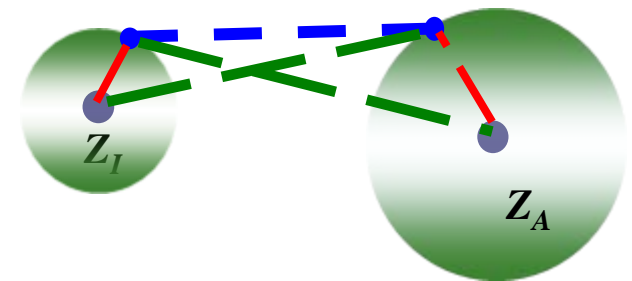
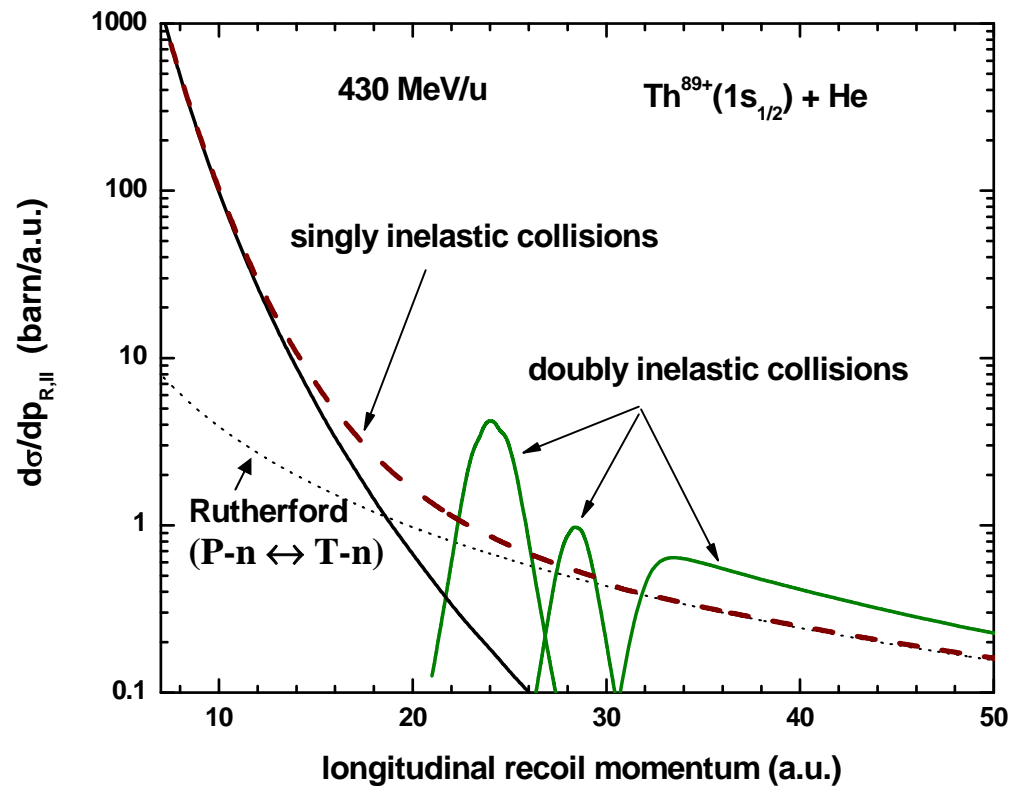
# Results: Target-recoil momentum spectra

## Singly inelastic collisions



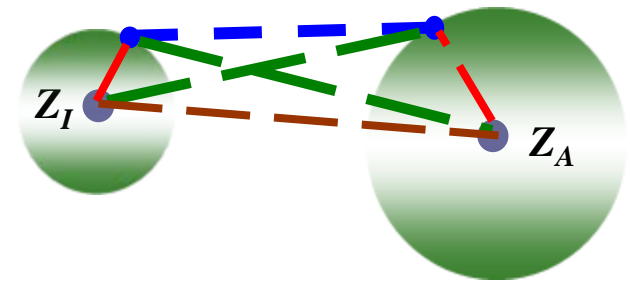
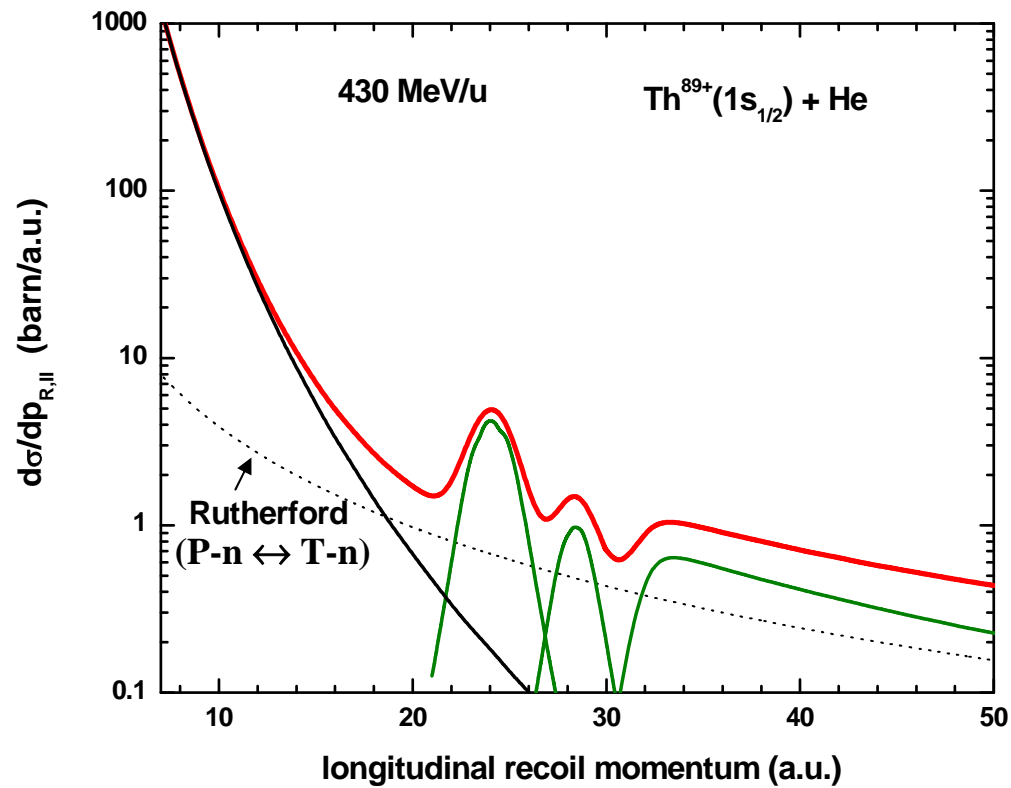
# Results: Target-recoil momentum spectra

## Singly/Doubly inelastic collisions



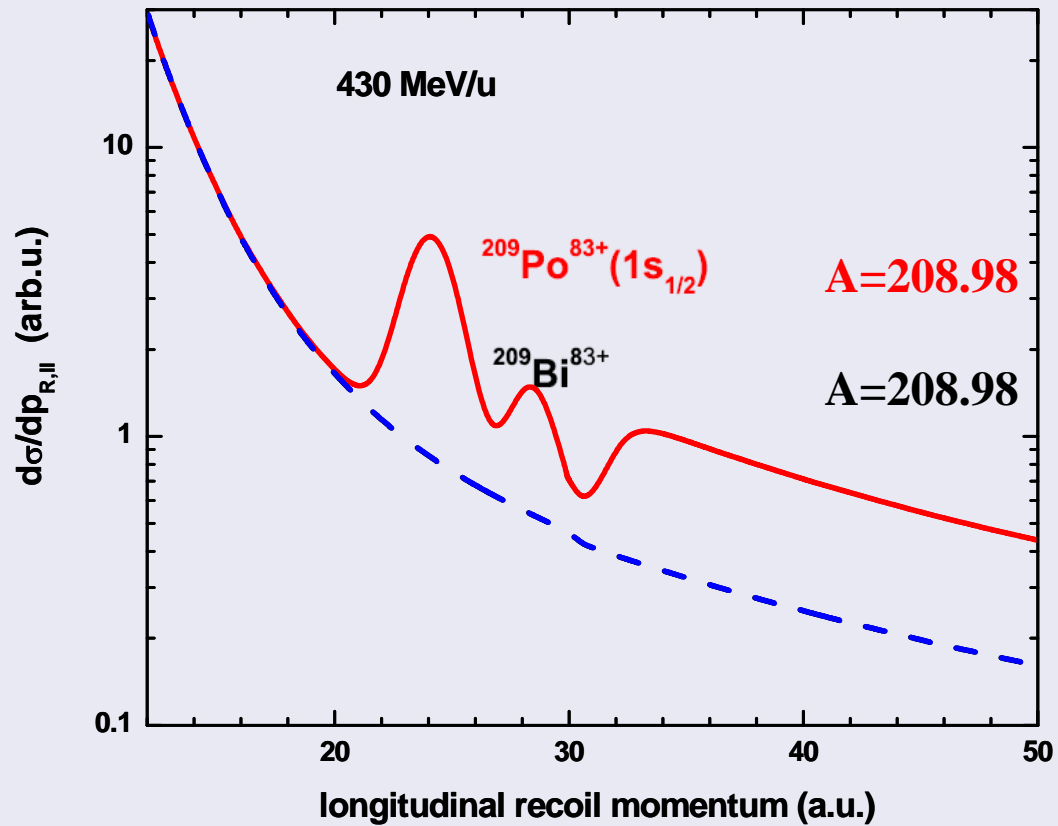
# Results

## Target-recoil momentum spectra



# Results

## Target-recoil momentum spectra





# Conclusions

Concentrating on the **longitudinal momentum spectrum** of target recoil ions we have considered the **inelastic collisions** of highly charged hydrogen-like ions with the simplest targets. We have shown that:

- the collision **dynamics is strongly influenced** by the **higher order effects** in the projectile-target interaction;
- the relativistic effects, caused both by the **relative ion-atom motion** and the electron motion in the **internal states** of the ion, are clearly manifested;
- the **projectile electron** is very well **visible** in the momentum transfer to the target recoil;
- **general picture has emerged** for the formation of the longitudinal momentum spectrum of the target recoil ion;
- at *moderate*  $\gamma$ 's, a **great deal** of **information** about the **doubly inelastic collisions** could be **obtained in experiment** by measuring just the longitudinal momentum spectrum of the target recoil ions.

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Thank you !

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$$\begin{aligned}
S_{fi}(\mathbf{Q}) = & -\frac{2i}{v} \int d^2\mathbf{p}_1 \int d^2\mathbf{p}_2 \int d^2\boldsymbol{\kappa} \\
& \frac{f(p_1, \nu) f(p_2, \nu) f(\kappa, -\eta)}{(\mathbf{q}^A + \boldsymbol{\kappa} - \mathbf{p}_1 - \mathbf{p}_2)^2 - \frac{(\varepsilon_n - \varepsilon_0)^2}{c^2}} \\
& \times \Phi_\mu^A (n0; \mathbf{q}^A + \boldsymbol{\kappa} - \mathbf{p}_1 - \mathbf{p}_2; \mathbf{p}_1; \mathbf{p}_2) \\
& \gamma^{-1} \Lambda_\alpha^\mu F_I^\alpha (m0; \mathbf{q}^I - \boldsymbol{\kappa} + \mathbf{p}_1 + \mathbf{p}_2)
\end{aligned}$$

$$f(a, \tau) = \frac{\Gamma(1 - i\tau)\Gamma(1/2 + i\tau)}{2\pi\Gamma(1/2)\Gamma(2i\tau)} a^{\delta - 2 + 2i\tau}$$

$$\begin{aligned}
\Phi_\mu^A \gamma^{-1} \Lambda_\alpha^\mu F_I^\alpha = & \left( \Phi_0^A + \frac{v}{c} \Phi_3^A \right) \left( F_I^0 + \frac{v}{c} F_I^3 \right) + \\
& \frac{\Phi_3^A F_I^3}{\gamma^2} + \frac{\Phi_1^A F_I^1 + \Phi_2^A F_I^2}{\gamma}
\end{aligned}$$

$$\Phi_0^A(n0; \mathbf{k}; \mathbf{p}_1; \mathbf{p}_2) = \langle \varphi_n | Z_A \exp(i\mathbf{p}_1 \cdot \mathbf{r}_1 + i\mathbf{p}_2 \cdot \mathbf{r}_2) - \exp(i\mathbf{p}_1 \cdot \mathbf{r}_1 + i\mathbf{p}_2 \cdot \mathbf{r}_2) \times (\exp(i\mathbf{k} \cdot \mathbf{r}_1) + \exp(i\mathbf{k} \cdot \mathbf{r}_2)) | \varphi_0 \rangle$$

$$\Phi_l^A(n0; \mathbf{k}; \mathbf{p}_1; \mathbf{p}_2) = \langle \varphi_n | \exp(i\mathbf{p}_1 \cdot \mathbf{r}_1 + i\mathbf{p}_2 \cdot \mathbf{r}_2) \times (\alpha_{l,1} \exp(i\mathbf{k} \cdot \mathbf{r}_1) + \alpha_{l,2} \exp(i\mathbf{k} \cdot \mathbf{r}_2)) | \varphi_0 \rangle,$$

$$F_0^I(m0; \mathbf{k}) = -\langle \chi_m | \exp(i\mathbf{k} \cdot \boldsymbol{\xi}) | \chi_0 \rangle$$

$$F_l^I(m0; \mathbf{k}) = \langle \chi_m | \alpha_l \exp(i\mathbf{k} \cdot \boldsymbol{\xi}) | \chi_0 \rangle$$

$$\gamma \quad q_{min}^A = \frac{\epsilon'_n - \epsilon'_0}{v_p} + \frac{\epsilon'_m - \epsilon'_0}{v_p}$$

$$q_{min}^A = \frac{\epsilon'_n - \epsilon'_0}{v_p} + \frac{\epsilon'_m - \epsilon'_0}{\gamma v_p}$$

$$q_{min}^A = \frac{\epsilon_n - \epsilon_0}{v_p} + \frac{\epsilon_m - \epsilon_0}{v_p}$$

$$q_{min}^A = \frac{\epsilon_n - \epsilon_0}{v_p} + \frac{\epsilon_m - \epsilon_0}{\gamma v_p}$$